19–1.

The rigid body (slab) has a mass $m$ and rotates with an angular velocity $\omega$ about an axis passing through the fixed point $O$. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude $mv_G$ and acting through point $P$, called the center of percussion, which lies at a distance $r_{PG} = k_G^2/r_{GO}$ from the mass center $G$. Here $k_G$ is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through $G$.

**SOLUTION**

\[ H_O = (r_{GO} + r_{PG}) mv_G = r_{GO} (mv_G) + I_G \omega, \quad \text{where} \quad I_G = mk_G^2 \]

\[ r_{GO} (mv_G) + r_{PG} (mv_G) = r_{GO} (mv_G) + (mk_G^2) \omega \]

\[ r_{PG} = \frac{k_G^2}{v_G \omega} \]

However, $v_G = \omega r_{GO}$ or $r_{GO} = \frac{v_G}{\omega}$

\[ r_{PG} = \frac{k_G^2}{r_{GO}} \]

Q.E.D.
19–2.

At a given instant, the body has a linear momentum \( \mathbf{L} = m \mathbf{v}_G \) and an angular momentum \( \mathbf{H}_G = I_G \omega \) computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity \( IC \) can be expressed as \( \mathbf{H}_{IC} = I_{IC} \omega \), where \( I_{IC} \) represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the \( IC \) is located at a distance \( r_{G/IC} \) away from the mass center \( G \).

**SOLUTION**

\[ H_{IC} = r_{G/IC} (mv_G) + I_G \omega, \quad \text{where} \quad \omega = \omega r_{G/IC} \]

\[ = r_{G/IC} (m \omega r_{G/IC}) + I_G \omega \]

\[ = (I_G + mr_{G/IC}^2) \omega \]

\[ = I_{IC} \omega \]

Q.E.D.
19–3.

Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center $G$, the angular momentum is the same when computed about any other point $P$.

**SOLUTION**

Since $v_G = 0$, the linear momentum $L = m v_G = 0$. Hence the angular momentum about any point $P$ is

$$H_P = I_G \omega$$

Since $\omega$ is a free vector, so is $H_P$. \[Q.E.D.\]
19–4.

The 40-kg disk is rotating at \( \omega = 100 \text{ rad/s} \). When the force \( P \) is applied to the brake as indicated by the graph. If the coefficient of kinetic friction at \( B \) is \( \mu_k = 0.3 \), determine the time \( t \) needed to stay the disk from rotating. Neglect the thickness of the brake.

**SOLUTION**

**Equilibrium.** Since slipping occurs at brake pad, \( F_f = \mu_k N = 0.3 \text{ N} \).

Referring to the FBD the brake's lever, Fig. a,

\[
\zeta + \sum M_A = 0; \quad N(0.6) - 0.3 \text{ N}(0.2) - P(0.3) = 0
\]

\[
N = 0.5556 \text{ P}
\]

Thus,

\[
F_f = 0.3(0.5556 \text{ P}) = 0.1667 \text{ P}
\]

**Principle of Impulse and Momentum.** The mass moment of inertia of the disk about its center \( O \) is \( I_O = \frac{1}{2} mr^2 = \frac{1}{2}(40)(0.15^2) = 0.45 \text{ kg \cdot m}^2 \).

\[
I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_O dt = I_O\omega_2
\]

It is required that \( \omega_2 = 0 \). Assuming that \( t > 2 \text{ s} \),

\[
0.45(100) + \int_0^t [-0.1667 \text{ P}(0.15)] dt = 0.45(0)
\]

\[
0.025 \int_0^t P \, dt = 45
\]

\[
\int_0^t P \, dt = 1800
\]

\[
\frac{1}{2}(500)(2) + 500(t - 2) = 1800
\]

\[
t = 4.60 \text{ s}
\]

Ans.

Since \( t > 2 \text{ s} \), the assumption was correct.

Ans: \( t = 4.60 \text{ s} \)
19–5.

The impact wrench consists of a slender 1-kg rod $AB$ which is 580 mm long, and cylindrical end weights at $A$ and $B$ that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to turn about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod $AB$ is given an angular velocity of $4 \text{ rad/s}$ and it strikes the bracket $C$ on the handle without rebounding, determine the angular impulse imparted to the lug nut.

**SOLUTION**

$$I_{\text{axle}} = \frac{1}{12}(1)(0.6 - 0.02)^2 + \frac{1}{2}(1)(0.01)^2 + 1(0.3)^2 = 0.2081 \text{ kg} \cdot \text{m}^2$$

$$\int M \, dt = I_{\text{axle}} \omega = 0.2081(4) = 0.833 \text{ kg} \cdot \text{m}^2/\text{s} \quad \text{Ans.}$$
19–6.

The airplane is traveling in a straight line with a speed of 300 km/h, when the engines $A$ and $B$ produce a thrust of $T_A = 40 \text{kN}$ and $T_B = 20 \text{kN}$, respectively. Determine the angular velocity of the airplane in $t = 5 \text{s}$. The plane has a mass of 200 Mg, its center of mass is located at $G$, and its radius of gyration about $G$ is $k_G = 15 \text{m}$.

**SOLUTION**

*Principle of Angular Impulse and Momentum:* The mass moment of inertia of the airplane about its mass center is $I_G = mk_G^2 = 200 \left(10^3\right) \left(15^2\right) = 45 \left(10^6\right) \text{kg} \cdot \text{m}^2$.

Applying the angular impulse and momentum equation about point $G$,

$$I_G \omega_1 + \sum \int_{t_1}^{t_2} M_G dt = I_G \omega_2$$

$$0 + 40 \left(10^3\right)(5)(8) - 20 \left(10^3\right)(5)(8) = 45 \left(10^6\right) \omega$$

$$\omega = 0.0178 \text{ rad/s} \quad \text{Ans.}$$

Ans:

$$\omega = 0.0178 \text{ rad/s}$$
19–7.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration of \( k_O = 110 \text{ mm} \). If the block at \( A \) has a mass of 40 kg, determine the speed of the block in 3 s after a constant force of 2 kN is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest.

**SOLUTION**

*Principle of Impulse and Momentum:* The mass moment inertia of the pulley about point \( O \) is \( I_O = 15(0.11^2) = 0.1815 \text{ kg} \cdot \text{m}^2 \). The angular velocity of the pulley and the velocity of the block can be related by \( \omega = \frac{v_B}{0.2} = 5v_B \). Applying Eq. 19–15, we have

\[
\left( \sum \text{syst. angular momentum} \right)_{O_1} + \left( \sum \text{syst. angular impulse} \right)_{O_1 \rightarrow 2} = \left( \sum \text{syst. angular momentum} \right)_{O_2}
\]

\[
\begin{align*}
(\zeta +) & \quad 0 + [40(9.81)(3)](0.2) - [2000(3)](0.075) \\
& \quad = -40v_B(0.2) - 0.1815(5v_B)
\end{align*}
\]

\[
v_B = 24.1 \text{ m/s}
\]

**Ans:**

\( v_B = 24.1 \text{ m/s} \)
**19–8.**

The assembly weighs 10 lb and has a radius of gyration \( k_G = 0.6 \text{ ft} \) about its center of mass \( G \). The kinetic energy of the assembly is 31 \( \text{ ft} \cdot \text{lb} \) when it is in the position shown. If it is rolling counterclockwise on the surface without slipping, determine its linear momentum at this instant.

**SOLUTION**

\[
I_G = (0.6)^2 \left( \frac{10}{32.2} \right) = 0.1118 \text{ slug} \cdot \text{ft}^2
\]

\[
T = \frac{1}{2} \left( \frac{10}{32.2} \right) v_G^2 + \frac{1}{2} (0.1118) \omega^2 = 31
\]

\[v_G = 1.2 \omega\]

Substitute into Eq. (1),

\[
\omega = 10.53 \text{ rad/s}
\]

\[
v_G = 10.53(1.2) = 12.64 \text{ ft/s}
\]

\[
L = m v_G = \frac{10}{32.2} (12.64) = 3.92 \text{ slug} \cdot \text{ft/s}
\]

**Ans:**

\[L = 3.92 \text{ slug} \cdot \text{ft/s}\]
19–9.

The disk has a weight of 10 lb and is pinned at its center O. If a vertical force of \( P = 2 \) lb is applied to the cord wrapped around its outer rim, determine the angular velocity of the disk in four seconds starting from rest. Neglect the mass of the cord.

**SOLUTION**

\[
I_O \omega_1 + \sum \int_{t_1}^{t_2} M_O \, dt = I_O \omega_2
\]

\[
0 + 2(0.5)(4) = \left[ \frac{1}{2} \left( \frac{10}{32.2} \right) (0.5)^2 \right] \omega_2
\]

\[
\omega_2 = 103 \text{ rad/s}
\]

Ans:

\[
\omega_2 = 103 \text{ rad/s}
\]
19–10.

The 30-kg gear A has a radius of gyration about its center of mass \(K = 125\) mm. If the 20-kg gear rack B is subjected to a force of \(P = 200\) N, determine the time required for the gear to obtain an angular velocity of 20 rad/s, starting from rest. The contact surface between the gear rack and the horizontal plane is smooth.

**SOLUTION**

**Kinematics:** Since the gear rotates about the fixed axis, the final velocity of the gear rack is required to be

\[
(v_B)_2 = \omega_B P = 20(0.15) = 3 \text{ m/s} \rightarrow
\]

**Principle of Impulse and Momentum:** Applying the linear impulse and momentum equation along the \(x\) axis using the free-body diagram of the gear rack shown in Fig. a,

\[
\begin{align*}
(m(v_B)_1 + \sum \int_{t_1}^{t_2} F_x dt) & = m(v_B)_2 \\
0 + 200(t) - F(t) & = 20(3)
\end{align*}
\]

\[
F(t) = 200t - 60 \tag{1}
\]

The mass moment of inertia of the gear about its mass center is \(I_O = mkO^2 = 30(0.125^2) = 0.46875 \text{ kg} \cdot \text{m}^2\). Writing the angular impulse and momentum equation about point \(O\) using the free-body diagram of the gear shown in Fig. b,

\[
\begin{align*}
I_O \omega_1 + \sum \int_{t_1}^{t_2} M_O dt & = I_O \omega_2 \\
0 + F(t)(0.15) & = 0.46875(20)
\end{align*}
\]

\[
F(t) = 62.5 \tag{2}
\]

Substituting Eq. (2) into Eq. (1) yields

\[
t = 0.6125 \text{ s}
\]

**Ans.:**

\(t = 0.6125 \text{ s}\)
19–11.

The pulley has a weight of 8 lb and may be treated as a thin disk. A cord wrapped over its surface is subjected to forces $T_A = 4$ lb and $T_B = 5$ lb. Determine the angular velocity of the pulley when $t = 4$ s if it starts from rest when $t = 0$. Neglect the mass of the cord.

**SOLUTION**

**Principle of Impulse and Momentum:** The mass moment inertia of the pulley about its mass center is $I_o = \frac{1}{2} \left( \frac{8}{32.2} \right) (0.6)^2 = 0.04472 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 19–14, we have

$$I_o \omega_1 + \sum \int_{t_1}^{t} M \omega \, dt = I_o \omega_2$$

$$\left( \sum \right) \quad 0 + [5(4)(0.6)] - [4(4)(0.6)] = 0.04472 \omega_2$$

$$\omega_2 = 53.7 \text{ rad/s} \quad \text{Ans.}$$
*19–12.

The 40-kg roll of paper rests along the wall where the coefficient of kinetic friction is \( \mu_k = 0.2 \). If a vertical force of \( P = 40 \text{ N} \) is applied to the paper, determine the angular velocity of the roll when \( t = 6 \text{ s} \) starting from rest. Neglect the mass of the unraveled paper and take the radius of gyration of the spool about the axle \( O \) to be \( k_O = 80 \text{ mm} \).

**SOLUTION**

**Principle of Impulse and Momentum.** The mass moment of inertia of the paper roll about its center is \( I_O = mk_O^2 = 40(0.08^2) = 0.256 \text{ kg} \cdot \text{m}^2 \). Since the paper roll is required to slip at point of contact, \( F_f = \mu_k N = 0.2 \text{ N} \). Referring to the FBD of the paper roll, Fig. a,

\[
0 + N(6) - F_{AB}\left(\frac{5}{13}\right)(6) = 0
\]

\[ F_{AB} = \frac{13}{5} \text{ N} \]

\[
0 + 0.2 \text{ N}(6) + F_{AB}\left(\frac{12}{13}\right)(6) + 40(6) - 40(9.81)(6) = 0
\]

\[
0.2 \text{ N} + \frac{12}{13} F_{AB} = 352.4
\]

Solving Eqs. (1) and (2)

\[ N = 135.54 \text{ N} \quad F_{AB} = 352.4 \text{ N} \]

Subsequently

\[
\zeta + I_O \omega_1 + \sum M_O dt = I_O \omega_2
\]

\[
0.256(0) + 40(0.12)(6) - 0.2(135.54)(0.12)(6) = 0.256 \omega
\]

\[ \omega = 36.26 \text{ rad/s} = 36.3 \text{ rad/s} \]

**Ans:**

\[ \omega = 36.3 \text{ rad/s} \]

The slender rod has a mass $m$ and is suspended at its end $A$ by a cord. If the rod receives a horizontal blow giving it an impulse $I$ at its bottom $B$, determine the location $y$ of the point $P$ about which the rod appears to rotate during the impact.

**SOLUTION**

**Principle of Impulse and Momentum:**

\[
I_G\omega_1 + \sum \int_{t_1}^{t_2} M_G \, dt = I_G\omega_2
\]

\[
0 + I \left( \frac{l}{2} \right) = \left[ \frac{1}{12}ml^2 \right] \omega \quad I = \frac{1}{6} ml\omega
\]

\[
I(v_{Ax})_1 + \sum \int_{t_1}^{t_2} F_x \, dt = I(v_{Ax})_2
\]

\[
0 + \frac{1}{6} ml\omega = mv_B \quad v_B = \frac{l}{6} \omega
\]

**Kinematics:** Point $P$ is the IC.

\[v_B = \omega y\]

Using similar triangles,

\[\frac{\omega y}{y} = \frac{l}{\omega} \quad y = \frac{2}{3} l\]

**Ans:**

\[y = \frac{2}{3} l\]
The rod of length $L$ and mass $m$ lies on a smooth horizontal surface and is subjected to a force $P$ at its end $A$ as shown. Determine the location $d$ of the point about which the rod begins to turn, i.e., the point that has zero velocity.

**SOLUTION**

(↓) \[ m(v_G)_1 + \sum F_x \, dt = m(v_G)_2 \]

0 + $P(t) = m(v_G)_x$

(+↑) \[ m(v_G)_1 + \sum F_y \, dt = m(v_G)_2 \]

0 + 0 = $m(v_G)_y$

(ζ+) \[ (H_G)_1 + \sum M_G \, dt = (H_G)_2 \]

0 + $P(t)\left(\frac{L}{2}\right) = \frac{1}{12} mL^2 \omega$

$v_G = y \omega$

$m(v_G)_x\left(\frac{L}{2}\right) = \frac{1}{12} mL^2 \omega$

$(v_G)_x = \frac{L}{6} \omega$

$y = \frac{L}{6}$

$d = \frac{L}{2} + \frac{L}{6} = \frac{2}{3} L$

**Ans:**

\[ d = \frac{2}{3} L \]
A 4-kg disk $A$ is mounted on arm $BC$, which has a negligible mass. If a torque of $M = (5e^{0.5t})$ N·m, where $t$ is in seconds, is applied to the arm at $C$, determine the angular velocity of $BC$ in 2 s starting from rest. Solve the problem assuming that (a) the disk is set in a smooth bearing at $B$ so that it rotates with curvilinear translation, (b) the disk is fixed to the shaft $BC$, and (c) the disk is given an initial freely spinning angular velocity of $\omega_B = \{-80\text{ rad/s}\}$ prior to application of the torque.

**SOLUTION**

a) 

\[(H_2) + \sum \int M_z\, dt = (H_2)\]

\[0 + \int_0^2 5e^{0.5t}\, dt = 4(v_B)(0.25)\]

\[\frac{5}{0.5}e^{0.5t}\bigg|_0^2 = v_B\]

\[v_B = 17.18\, \text{m/s}\]

Thus,

\[\omega_{BC} = \frac{17.18}{0.25} = 68.7\, \text{rad/s}\]

**Ans.**

b) 

\[(H_2) + \sum \int M_z\, dt = (H_2)\]

\[0 + \int_0^2 5e^{0.5t}\, dt = 4(v_B)(0.25) + \left[\frac{1}{2}(4)(0.06)^2\right]\omega_{BC}\]

Since $v_B = 0.25\, \omega_{BC}$, then

\[\omega_{BC} = 66.8\, \text{rad/s}\]

**Ans.**

c) 

\[(H_2) + \sum \int M_z\, dt = (H_2)\]

\[-\left[\frac{1}{2}(4)(0.06)^2\right](80) + \int_0^2 5e^{0.5t}\, dt = 4(v_B)(0.25) - \left[\frac{1}{2}(4)(0.06)^2\right](80)\]

Since $v_B = 0.25\, \omega_{BC}$,

\[\omega_{BC} = 68.7\, \text{rad/s}\]

**Ans.**

**Ans:**

(a) $\omega_{BC} = 68.7\, \text{rad/s}$
(b) $\omega_{BC} = 66.8\, \text{rad/s}$
(c) $\omega_{BC} = 68.7\, \text{rad/s}$
The frame of a tandem drum roller has a weight of 4000 lb excluding the two rollers. Each roller has a weight of 1500 lb and a radius of gyration about its axle of 1.25 ft. If a torque of \( M = 300 \text{ lb-ft} \) is supplied to the rear roller \( A \), determine the speed of the drum roller 10 s later, starting from rest.

**Solution**

**Principle of Impulse and Momentum:** The mass moments of inertia of the rollers about their mass centers are \( I_c = I_p = \frac{1500}{32.2} \left( 1.25^2 \right) = 72.787 \text{ slug-ft}^2 \). Since the rollers roll without slipping, \( \omega = \frac{v}{r} = \frac{v}{1.5} = 0.6667v \). Using the free-body diagrams of the rear roller and front roller, Figs. a and b, and the momentum diagram of the rollers, Fig. c,

\[
(H_A)_1 + \sum \int_{t_1}^{t_2} M_A dt = (H_A)_2
\]

\[
0 + 300(10) - C_x(10)(1.5) = \frac{1500}{32.2}v(1.5) + 72.787(0.6667v)
\]

\[
C_x = 200 - 7.893v 
\]

and

\[
(H_B)_1 + \sum \int_{t_1}^{t_2} M_B dt = (H_B)_2
\]

\[
0 + D_x(10)(1.5) = \frac{1500}{32.2}v(1.5) + 72.787(0.6667v)
\]

\[
D_x = 7.893v 
\]

Referring to the free-body diagram of the frame shown in Fig. d,

\[
m \left[ (v_G)_{1x} \right]_1 + \sum \int_{t_1}^{t_2} F_x dt = m \left[ (v_G)_{2x} \right]_2
\]

\[
0 + C_x(10) - D_A(10) = \frac{4000}{32.2}v 
\]

Substituting Eqs. (1) and (2) into Eq. (3),

\[
(200 - 7.893v)(10) - 7.893v(10) = \frac{4000}{32.2}v
\]

\[
v = 7.09 \text{ ft/s}
\]

**Ans:**

\[
v = 7.09 \text{ ft/s}
\]
19–17.

The 100-lb wheel has a radius of gyration of \( k_G = 0.75 \) ft. If the upper wire is subjected to a tension of \( T = 50 \) lb, determine the velocity of the center of the wheel in 3 s, starting from rest. The coefficient of kinetic friction between the wheel and the surface is \( \mu_k = 0.1 \).

**SOLUTION**

**Principle of Impulse and Momentum:** We can eliminate the force \( F \) from the analysis if we apply the principle of impulse and momentum about point \( A \). The mass moment inertia of the wheel about point \( A \) is \( I_A = \frac{100}{32.2} (0.75^2) + \frac{100}{32.2} (0.5^2) = 2.523 \text{ slug} \cdot \text{ft}^2 \). Applying Eq. 19–14, we have

\[
m(v_G)_1 + \sum \int_{t_1}^{t_2} T_j dt = m(v_G)_2
\]

\[(+ \uparrow) \quad 0 + N(t) - 100(t) = 0 \quad N = 100 \text{ lb} \]

\[
I_A \omega_1 + \sum \int_{t_1}^{t_2} M_A dt = I_A \omega_2
\]

\[(- \uparrow) \quad 0 + [50(3)](1) - [0.1(100)(3)][0.5] = 2.523 \omega_2 \quad [1]
\]

**Kinematics:** Since the wheel rolls without slipping at point \( A \), the instantaneous center of zero velocity is located at point \( A \). Thus,

\[
v_G = \omega_2 r_{GIC}
\]

\[
\omega_2 = \frac{v_G}{r_{GIC}} = \frac{v_G}{0.5} = 2v_G \quad [2]
\]

Solving Eqs. [1] and [2] yields

\[
v_G = 26.8 \text{ ft/s} \quad \text{Ans.}
\]

\[
\omega_2 = 53.50 \text{ rad/s}
\]
The 4-kg slender rod rests on a smooth floor. If it is kicked so as to receive a horizontal impulse \( I = 8 \, \text{N} \cdot \text{s} \) at point \( A \) as shown, determine its angular velocity and the speed of its mass center.

**SOLUTION**

\[
\begin{align*}
\text{Ans.} & \quad v_G = 2 \, \text{m/s} \\
\text{Ans.} & \quad \omega = 3.90 \, \text{rad/s}
\end{align*}
\]
19–19.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration \( k_O = 110 \text{ mm} \). If the block at \( A \) has a mass of 40 kg, determine the speed of the block in 3 s after a constant force \( F = 2 \text{ kN} \) is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.

**SOLUTION**

\[
(C + ) \quad (H_O)_1 + \sum M_O \, dt = (H_O)_2
\]

\[
0 + 2000(0.075)(3) - 40(9.81)(0.2)(3) = 15(0.110)^2 \omega + 40(0.2\omega) (0.2)
\]

\[
\omega = 120.4 \text{ rad/s}
\]

\[
v_A = 0.2(120.4) = 24.1 \text{ m/s}
\]

**Ans:**

\[
v_A = 24.1 \text{ m/s}
\]
The 100-kg spool is resting on the inclined surface for which the coefficient of kinetic friction is \( \mu_k = 0.1 \). Determine the angular velocity of the spool when \( t = 4 \text{ s} \) after it is released from rest. The radius of gyration about the mass center is \( k_G = 0.25 \text{ m} \).

**SOLUTION**

**Kinematics.** The IC of the spool is located as shown in Fig. a. Thus

\[
v_G = \omega r_{G/IC} = \omega(0.2)
\]

**Principle of Impulse and Momentum.** The mass moment of inertia of the spool about its mass center is \( I_G = mk_G^2 = 100(0.25^2) = 6.25 \text{ kg} \cdot \text{m}^2 \). Since the spool is required to slip, \( F_f = \mu_k N = 0.1 \text{ N} \). Referring to the FBD of the spool, Fig. b,

\[
\begin{align*}
\begin{aligned}
\sum F_y & = m(v_G)_y_1 + \sum F_y \int_{t_1}^{t_2} dt = m(v_G)_y_2 \\
0 + N(4) - 100(9.81) \cos 30^\circ(4) &= 0 \\
N &= 849.57 \text{ N}
\end{aligned} \\
\sum L_x & = m(v_G)_x_1 + \sum L_x \int_{t_1}^{t_2} dt = m(v_G)_x_2 \\
0 + T(4) + 0.1(849.57)(4) - 100(9.81) \sin 30^\circ(4) &= 100[ - \omega(0.2)] \\
T + 5\omega &= 405.54 \\
\end{align*}
\]

\[
\begin{align*}
\begin{aligned}
\sum M_G & = I_G \omega_1 + \sum M_G \int_{t_1}^{t_2} dt = I_G \omega_2 \\
0 + 0.1(849.57)(0.4)(4) - T(0.2)(4) &= -6.25 \omega_2 \\
0.8T - 6.25\omega_2 &= 135.93
\end{aligned}
\end{align*}
\]

Solving Eqs. (1) and (2),

\[
\begin{align*}
\omega &= 18.39 \text{ rad/s} = 18.4 \text{ rad/s} \\
T &= 313.59 \text{ N}
\end{align*}
\]

\textbf{Ans.}

\[
\begin{align*}
\omega &= 18.4 \text{ rad/s} \\
T &= 313.59 \text{ N}
\end{align*}
\]
19–21.

The spool has a weight of 30 lb and a radius of gyration \( k_O = 0.45 \) ft. A cord is wrapped around its inner hub and the end subjected to a horizontal force \( P = 5 \) lb. Determine the spool’s angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.

**SOLUTION**

\[ (+i) \quad m(v_y) + \sum F_y \, dt = m(v_y)_2 \]

\[ 0 + N_A(4) - 30(4) = 0 \]

\[ N_A = 30 \text{ lb} \]

\[ (-i) \quad m(v_x) + \sum F_x \, dt = m(v_x)_2 \]

\[ 0 + 5(4) - F_A(4) = \frac{30}{32.2} v_G \]

\[ (c+) \quad (H_G)_1 + \sum M_G \, dt = (H_G)_2 \]

\[ 0 + F_A(4)(0.9) - 5(4)(0.3) = \frac{30}{32.2}(0.45)^2 \omega \]

Since no slipping occurs

Set \( v_G = 0.9 \omega \)

\( F_A = 2.33 \text{ lb} \)

\[ \omega = 12.7 \text{ rad/s} \quad \text{Ans.} \]

Also,

\[ (c+) \quad (H_A)_1 + \sum M_A \, dt = (H_A)_2 \]

\[ 0 + 5(4)(0.6) = \left[ \frac{30}{32.2}(0.45)^2 + \frac{30}{32.2}(0.9)^2 \right] \omega \]

\[ \omega = 12.7 \text{ rad/s} \quad \text{Ans.} \]
The two gears $A$ and $B$ have weights and radii of gyration of $W_A = 15$ lb, $k_A = 0.5$ ft and $W_B = 10$ lb, $k_B = 0.35$ ft, respectively. If a motor transmits a couple moment to gear $B$ of $M = 2(1 - e^{-0.5t})$ lb-ft, where $t$ is in seconds, determine the angular velocity of gear $A$ in $t = 5$ s, starting from rest.

**SOLUTION**

$\omega_A(0.8) = \omega_B(0.5)$

$\omega_B = 1.6\omega_A$

Gear $B$:

$$(C +) \quad (H_B)_1 + \sum M_B \, dt = (H_B)_2$$

$$0 + \int_0^5 2(1 - e^{-0.5t}) \, dt - \int_0^5 0.5F \, dt = \left[ \left( \frac{10}{32.2} \right)(0.35)^2 \right](1.6\omega_A)$$

$$6.328 = 0.5 \int F \, dt + 0.06087\omega_A \quad (1)$$

Gear $A$:

$$0 = 0.8 \int F \, dt - 0.1165\omega_A \quad (2)$$

$$(C +) \quad (H_A)_1 + \sum M_A \, dt = (H_A)_2$$

Eliminate $\int F \, dt$ between Eqs. (1) and (2), and solving for $\omega_A$.

$$0 + \int 0.8F \, dt = \left[ \left( \frac{15}{32.2} \right)(0.5)^2 \right]\omega_A \quad \omega_A = 47.3 \, \text{rad/s} \quad \text{Ans.}$$

Ans:

$$\omega_A = 47.3 \, \text{rad/s}$$
19–23.

The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin \( \omega = 8 \text{ rad/s} \) and its center has a velocity \( v_G = 3 \text{ m/s} \) as shown. If the coefficient of kinetic friction between the hoop and the plane is \( \mu_k = 0.6 \), determine how long the hoop rolls before it stops slipping.

**SOLUTION**

\[
\begin{align*}
\mathcal{J} + mv_{t_1} + \sum F_x dt &= mv_{t_2} \\
5(3) + 49.05 \sin 30^\circ (t) - 25.487t &= 5v_G \\
\zeta + (H_G)_1 + \sum M_G dt &= (H_G)_2 \\
-5(0.5)^2(8) + 25.487(0.5)(t) &= 5(0.5)^2 \frac{v_G}{0.5}
\end{align*}
\]

Solving,

\[
\begin{align*}
v_G &= 2.75 \text{ m/s} \\
t &= 1.32 \text{ s}
\end{align*}
\]

Ans.

Ans: 

\( t = 1.32 \text{ s} \)
The 30-kg gear is subjected to a force of $P = (20t)$ N, where $t$ is in seconds. Determine the angular velocity of the gear at $t = 4$ s, starting from rest. Gear rack $B$ is fixed to the horizontal plane, and the gear’s radius of gyration about its mass center $O$ is $k_O = 125$ mm.

**SOLUTION**

**Kinematics:** Referring to Fig. $a$,

$$v_O = \omega r_O/IC = \omega (0.15)$$

**Principle of Angular Impulse and Momentum:** The mass moment of inertia of the gear about its mass center is $I_O = mk_O^2 = 30(0.125^2) = 0.46875 \text{ kg} \cdot \text{m}^2$. Writing the angular impulse and momentum equation about point $A$ shown in Fig. $b$,

$$(H_A)_1 + \sum \int_{t_1}^{t_2} M_A dt = (H_A)_2$$

$$0 + \int_{0}^{4s} 20t(0.15)dt = 0.46875\omega + 30[\omega(0.15)](0.15)$$

$$1.5t^2 \bigg|_0^{4s} = 1.14375\omega$$

$$\omega = 21.0 \text{ rad/s}$$

**Ans.**

$$\omega = 21.0 \text{ rad/s}$$
The 30-lb flywheel $A$ has a radius of gyration about its center of 4 in. Disk $B$ weighs 50 lb and is coupled to the flywheel by means of a belt which does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque to the flywheel of $M = (50t)$ lb-ft, where $t$ is in seconds, determine the time required for the disk to attain an angular velocity of 60 rad/s starting from rest.

**SOLUTION**

**Principle of Impulse and Momentum:** The mass moment inertia of the flywheel about point $C$ is $I_C = \frac{30}{32.2} \left(\frac{4}{12}\right)^2 = 0.1035$ slug $\cdot$ ft$^2$. The angular velocity of the flywheel is $\omega_A = \frac{r_B}{r_A} \omega_B = \frac{0.75}{0.5} (60) = 90.0$ rad/s. Applying Eq. 19–14 to the flywheel [FBD(a)], we have

\[
I_C \omega_1 + \int_{t_1}^{t_2} M_C \, dt = I_C \omega_2
\]

\[
(\zeta+) \quad 0 + \int_0^t 50t \, dt + \left[ \int T_2 (dt) \right] (0.5) - \left[ \int T_1 (dt) \right] (0.5) = 0.1035(90)
\]

\[
25t^2 + 0.5 \int (T_2 - T_1) \, dt = 9.317 \tag{1}
\]

The mass moment inertia of the disk about point $D$ is $I_D = \frac{1}{2} \left(\frac{50}{32.2}\right) (0.75^2) = 0.4367$ slug $\cdot$ ft$^2$. Applying Eq. 19–14 to the disk [FBD(b)], we have

\[
I_D \omega_1 + \int_{t_1}^{t_2} M_D \, dt = I_D \omega_2
\]

\[
(\zeta+) \quad 0 + \left[ \int T_1 (dt) \right] (0.75) - \left[ \int T_2 (dt) \right] (0.75) = 0.4367(60)
\]

\[
\int (T_2 - T_1) \, dt = -34.94 \tag{2}
\]

Substitute Eq. (2) into Eq. (1) and solving yields

\[ t = 1.04 \text{ s} \]

**Ans:**

\[ t = 1.04 \text{ s} \]

If the shaft is subjected to a torque of \( M = (15t^2) \) N \cdot m, where \( t \) is in seconds, determine the angular velocity of the assembly when \( t = 3 \) s, starting from rest. Rods \( AB \) and \( BC \) each have a mass of 9 kg.

**SOLUTION**

*Principle of Impulse and Momentum:* The mass moment of inertia of the rods about their mass center is \( I_G = \frac{1}{12} ml^2 = \frac{1}{12} (9)(1^2) = 0.75 \text{ kg} \cdot \text{m}^2 \). Since the assembly rotates about the fixed axis, \((v_G)_{AB} = \omega(r_G)_{AB} = \omega(0.5)\) and \((v_G)_{BC} = \omega(r_G)_{BC} = \omega\left(\sqrt{1^2 + (0.5)^2}\right) = \omega(1.118)\). Referring to Fig. \( a \),

\[
\begin{align*}
\zeta + (H_2) &= \sum \int_{t_1}^{t_2} M_z \, dt = (H_2)_2 \\
0 + \int_0^{3s} 15t^2 \, dt &= 9 \left[ \omega(0.5) \right](0.5) + 0.75\omega + 9 \left[ \omega(1.118) \right](1.118) + 0.75\omega \\
5t^3 \bigg|_0^{3s} &= 15\omega \\
\omega &= 9 \text{ rad/s} \\
\text{Ans.}
\end{align*}
\]
19–27.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration of $k_O = 110$ mm. If the block at $A$ has a mass of 40 kg and the container at $B$ has a mass of 85 kg, including its contents, determine the speed of the container when $t = 3$ s after it is released from rest.

**SOLUTION**

The angular velocity of the pulley can be related to the speed of container $B$ by $\omega = \frac{v_A}{0.075} = 13.33\, v_B$. Also the speed of block $A$ $v_A = \omega(0.2) = 13.33\, v_B(0.2) - 2.667\, v_B$.

\[
(\Sigma\text{ Ang. Mom.})_{O1} + (\Sigma\text{ Ang. Imp.})_{O(1-2)} = (\Sigma\text{ Syst. Ang. Mom.})_{O2}
\]

\[
0 + 40(9.81)(0.2)(3) - 85(9.81)(0.075)(3) = [15(0.110)^2](13.33\, v_B) + 85\, v_B(0.075) + 40(2.667\, v_B)(0.2)
\]

$v_B = 1.59\, \text{m/s}$

Ans.
*19–28.

The crate has a mass $m_c$. Determine the constant speed $v_0$ it acquires as it moves down the conveyor. The rollers each have a radius of $r$, mass $m$, and are spaced $d$ apart. Note that friction causes each roller to rotate when the crate comes in contact with it.

**SOLUTION**

The number of rollers per unit length is $1/d$.

Thus in one second, $\frac{v_0}{d}$ rollers are contacted.

If a roller is brought to full angular speed of $\omega = \frac{v_0}{r}$ in $t_0$ seconds, then the moment of inertia that is effected is

$$I' = I \left(\frac{v_0}{d}\right) t_0 = \left(\frac{1}{2}mr^2\right) \left(\frac{v_0}{d}\right) t_0$$

Since the frictional impulse is

$$F = m_c \sin \theta$$

then

$$\zeta + (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$$

and

$$0 + (m_c \sin \theta) rt_0 = \left[ \left(\frac{1}{2}mr^2\right) \left(\frac{v_0}{d}\right) t_0 \right] \left(\frac{v_0}{r}\right)$$

$$v_0 = \sqrt{2 g \sin \theta \, d \left(\frac{m_c}{m}\right)}$$

Ans:
The turntable $T$ of a record player has a mass of 0.75 kg and a radius of gyration $k_z = 125$ mm. It is turning freely at $\omega_T = 2$ rad/s when a 50-g record (thin disk) falls on it. Determine the final angular velocity of the turntable just after the record stops slipping on the turntable.

**SOLUTION**

\[
(Hz)_1 = (Hz)_2 \\
0.75(0.125)^2(2) = 0.75(0.125)^2 + \frac{1}{2}(0.050)(0.150)^2 \omega \\
\omega = 1.91 \text{ rad/s} \quad \text{Ans.}
\]
The 10-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded into its edge. Also, calculate the angle \( \theta \) the disk will swing when it stops. The disk is originally at rest. Neglect the mass of the rod \( AB \).

**SOLUTION**

**Conservation of Angular Momentum.** The mass moment of inertia of the disk about its mass center is \( (I_G)_d = \frac{1}{2} (5)(0.4^2) = 0.4 \text{ kg} \cdot \text{m}^2 \). Also, \( (v_b)_2 = \omega_2 (\sqrt{5.92}) \) and \( (v_d)_2 = \omega_2 (2.4) \). Referring to the momentum diagram with the embedded bullet, Fig. a,

\[
\Sigma (H_A)_1 = \Sigma (H_A)_2
\]

\[
M_d (v_b)_2 (r_b)_1 = (I_G)_d \omega_2 + M_d (v_d)_2 (r_d)_2
\]

\[
0.01(800)(2.4) = 0.4 \omega_2 + 5 (\omega_2 (2.4)) (2.4) + 0.01 (\omega_2 (\sqrt{5.92})) (\sqrt{5.92})
\]

\[\omega_2 = 0.656 \text{ rad/s} \]

Ans.

**Kinetic Energy.** Since the system is required to stop finally, \( T_3 = 0 \). Here

\[
T_2 = \frac{1}{2} (I_G)_d \omega_2^2 + \frac{1}{2} M_d (v_d)_2^2 + \frac{1}{2} M_b (v_b)_2^2
\]

\[
= \frac{1}{2} (0.4)(0.6562^2) + \frac{1}{2}(5)(0.6562(2.4))^2 + \frac{1}{2}(0.01)(0.6562(\sqrt{5.92}))^2
\]

\[= 6.2996 \text{ J} \]

**Potential Energy.** Datum is set as indicated on Fig. b.

Here \( \phi = \tan^{-1} \left( \frac{0.4}{2.4} \right) = 9.4623^\circ \). Hence

\[y_1 = 2.4 \cos \theta \quad y_b = \sqrt{5.92} \cos (\theta - 9.4623^\circ)\]

Thus, the gravitational potential energy of the disk and bullet with reference to the datum is

\[
(V_d)_d = M_d g (y_d) = 5(9.81)(-2.4 \cos \theta) = -117.72 \cos \theta
\]

\[
(V_b)_b = M_b g (y_b) = 0.01(9.81)[(-\sqrt{5.92} \cos (\theta - 9.4623^\circ))]
\]

\[= -0.0981 \sqrt{5.92} \cos (\theta - 9.4623^\circ)\]

At \( \theta = 0^\circ \),

\[
|(V_d)_d| = -117.72 \cos 0^\circ = -117.72 \text{ J}
\]

\[
|(V_b)_b| = -0.0981 \sqrt{5.92} \cos (0 - 9.4623^\circ) = -0.23544 \text{ J}
\]

**Conservation of Energy.**

\[
T_2 + V_2 = T_3 + V_3
\]

\[
6.2996 + (-117.72) + (-0.23544) = 0 + (-117.72 \cos \theta)
\]

\[
+ \left[ -0.0981 \sqrt{5.92} \cos (\theta - 9.4623^\circ) \right]
\]

\[117.72 \cos \theta + 0.0981 \sqrt{5.92} \cos (\theta - 9.4623^\circ) = 111.66\]

Solved by numerically,

\[\theta = 18.83^\circ = 18.8^\circ \]

Ans.

\[\omega_2 = 0.656 \text{ rad/s} \]

\[\theta = 18.8^\circ \]
The 10-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded into its edge. Also, calculate the angle \( \theta \) the disk will swing when it stops. The disk is originally at rest. The rod \( AB \) has a mass of 3 kg.

**SOLUTION**

**Conservation of Angular Momentum.** The mass moments of inertia of the disk and rod about their respective mass centers are \((I_G)_d = \frac{1}{2}(5)(0.4^2) = 0.4 \text{ kg} \cdot \text{m}^2\) and \((I_G)_r = \frac{1}{12}(3)(2^2) = 1.00 \text{ kg} \cdot \text{m}^2\). Also, \((v_b)_1 = \omega_2(\sqrt{5.92})\), \((v_d)_2 = \omega_2(2.4)\) and \((v_r)_2 = \omega_2 (1)\). Referring to the momentum diagram with the embedded bullet, Fig. a,

\[
\Sigma (H_A)_1 = \Sigma (H_A)_2
\]

\[
M_b(v_b)_1(r_b)_1 = (I_G)_d \omega_2 + M_d(v_d)_2(r_d) + (I_G)_r \omega_2 + m_r(v_r)_2(r) + M_b(v_b)_2(r_b)_2
\]

\[
0.01(800)(2.4) = 0.4\omega_2 + 5[\omega_2(2.4)](2.4) + 1.00\omega_2 + 3[\omega_2(1)] (1)
\]

\[
\omega_2 = 0.5773 \text{ rad/s} = 0.577 \text{ rad/s}
\]

**Ans.**

**Kinetic Energy.** Since the system is required to stop finally, \( T_1 = 0 \). Here

\[
T_2 = \frac{1}{2}(I_G)_d \omega_2^2 + \frac{1}{2} M_d(v_d)_2^2 + \frac{1}{2} (I_G)_r \omega_2^2 + \frac{1}{2} M_r(v_r)_2^2 + \frac{1}{2} M_b(v_b)_2^2
\]

\[
= \frac{1}{2}(0.4)(0.5773^2) + \frac{1}{2}(5)[0.5773(2.4)]^2 + \frac{1}{2}(1.00)(0.5773^2) + \frac{1}{2}(3)[0.5773(1)]^2 + \frac{1}{2}(0.01)[0.5773(\sqrt{5.92})]^2
\]

\[
= 5.5419 \text{ J}
\]
19–31. Continued

**Potential Energy.** Datum is set as indicated on Fig. b.

Here \( \phi = \tan^{-1} \left( \frac{0.4}{2.4} \right) = 9.4623^\circ \). Hence

\[ y_d = 2.4 \cos \theta, \quad y_r = \cos \theta, \quad y_b = \sqrt{5.92} \cos (\theta - 9.4623^\circ) \]

Thus, the gravitational potential energy of the disk, rod and bullet with reference to the datum is

\[
(V^g)_d = M_d g y_d = 5(9.81)(-2.4 \cos \theta) = -117.72 \cos \theta
\]
\[
(V^g)_r = M_r g y_r = 3(9.81)(-\cos \theta) = -29.43 \cos \theta
\]
\[
(V^g)_b = m_b g y_b = 0.01(9.81) \left[ -\sqrt{5.92} \cos (\theta - 94623^\circ) \right]
\]
\[
= -0.0981 \sqrt{5.92} \cos (\theta - 9.4623^\circ)
\]

At \( \theta = 0^\circ \),

\[
[(V^g)_d]_1 = -117.72 \cos 0^\circ = -117.72 \text{ J}
\]
\[
[(V^g)_r]_1 = -29.43 \cos 0^\circ = -29.43 \text{ J}
\]
\[
[(V^g)_b]_2 = -0.0981 \sqrt{5.92} \cos (0^\circ - 9.4623^\circ) = -0.23544 \text{ J}
\]

**Conservation of Energy.**

\[
T_2 + V_2 = T_3 + V_3
\]
\[
5.5419 + (-117.72) + (-29.43) + (-0.23544) = 0 + (-117.72 \cos \theta)
\]
\[
+ (-29.43 \cos \theta) + \left[ -0.0981 \sqrt{5.92} \cos (\theta - 9.4623^\circ) \right]
\]
\[
147.15 \cos \theta + 0.0981 \sqrt{5.92} \cos (\theta - 9.4623^\circ) = 141.84
\]

Solved numerically,

\[
\theta = 15.78^\circ = 15.8^\circ \quad \text{Ans.}
\]

**Ans:**

\[
\omega_2 = 0.577 \text{ rad/s}
\]
\[
\theta = 15.8^\circ
\]
The circular disk has a mass \( m \) and is suspended at \( A \) by the wire. If it receives a horizontal impulse \( I \) at its edge \( B \), determine the location \( y \) of the point \( P \) about which the disk appears to rotate during the impact.

**SOLUTION**

**Principle of Impulse and Momentum.** The mass moment of inertia of the disk about its mass center is \( I_G = \frac{1}{2} mr^2 = \frac{1}{2} ma^2 \)

\[
\zeta + I_G \omega_1 + \sum \int_{t_1}^{t_2} M_G dt = I_G \omega_2
\]

\[
0 + I_a = \left( \frac{1}{2} ma^2 \right) \omega
\]

\[
I = \frac{1}{2} ma \omega \quad (1)
\]

\[
(\pm a) \quad m \left[ (v_{G})_x \right]_2 + \sum \int_{t_1}^{t_2} F_x dt = m \left[ (v_{G})_x \right]_2
\]

\[
0 + I = mv_G
\]

Equating Eqs. (1) and (2),

\[
\frac{1}{2} ma \omega = mv_G
\]

\[
v_G = \frac{a}{2} \omega
\]

**Kinematics.** Here, \( IC \) is located at \( P \), Fig. \( b \). Thus, \( v_B = \omega r_{B/IC} = \omega (2a - y) \). Using similar triangles,

\[
\frac{a - y}{v_G} = \frac{2a - y}{v_B}; \quad \frac{a - y}{a} = \frac{2a - y}{\omega(2a - y)}
\]

\[
y = \frac{1}{2} a
\]

Ans.
19–33.

The 80-kg man is holding two dumbbells while standing on a turntable of negligible mass, which turns freely about a vertical axis. When his arms are fully extended, the turntable is rotating with an angular velocity of 0.5 rev/s. Determine the angular velocity of the man when he retracts his arms to the position shown. When his arms are fully extended, approximate each arm as a uniform 6-kg rod having a length of 650 mm, and his body as a 68-kg solid cylinder of 400-mm diameter. With his arms in the retracted position, assume the man as an 80-kg solid cylinder of 450-mm diameter. Each dumbbell consists of two 5-kg spheres of negligible size.

**SOLUTION**

**Conservation of Angular Momentum:** Since no external angular impulse acts on the system during the motion, angular momentum about the axis of rotation (z axis) is conserved. The mass moment of inertia of the system when the arms are in the fully extended position is

\[
(I_1)_1 = 2 \left[ 10(0.85^2) \right] + 2 \left[ \frac{1}{12} (6)(0.65^2) + 6(0.525^2) \right] + \frac{1}{2} (68)(0.2^2)
\]

\[
= 19.54 \text{ kg} \cdot \text{m}^2
\]

And the mass moment of inertia of the system when the arms are in the retracted position is

\[
(I_2)_2 = 2 \left[ 10(0.3^2) \right] + \frac{1}{2} (80)(0.225^2)
\]

\[
= 3.825 \text{ kg} \cdot \text{m}^2
\]

Thus,

\[
(H_2)_1 = (H_2)_2
\]

\[
(I_1)_1\omega_1 = (I_2)_2\omega_2
\]

\[
19.54(0.5) = 3.825\omega_2
\]

\[
\omega_2 = 2.55 \text{ rev/s}
\]

Ans: \(\omega_2 = 2.55 \text{ rev/s}\)
The platform swing consists of a 200-lb flat plate suspended by four rods of negligible weight. When the swing is at rest, the 150-lb man jumps off the platform when his center of gravity $G$ is 10 ft from the pin at $A$. This is done with a horizontal velocity of 5 ft/s, measured relative to the swing at the level of $G$. Determine the angular velocity he imparts to the swing just after jumping off.

**SOLUTION**

\[
(H_A)_1 = (H_A)_2
\]

\[
0 + 0 = \left[ \frac{1}{12} \left( \frac{200}{32.2} \right) (4)^2 + \frac{200}{32.2} (11)^2 \right] \omega - \left[ \left( \frac{150}{32.2} \right) (5 - 10\omega) \right]
\]

\[
\omega = 0.190 \text{ rad/s}
\]

Ans: $\omega = 0.190 \text{ rad/s}$
The 2-kg rod $ACB$ supports the two 4-kg disks at its ends. If both disks are given a clockwise angular velocity $(\omega_A)_1 = (\omega_B)_1 = 5 \text{ rad/s}$ while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins $A$ and $B$. Motion is in the horizontal plane. Neglect friction at pin $C$.

SOLUTION

$$\zeta + H_1 = H_2$$

$$2 \left[ \frac{1}{2} (4)(0.15)^2 \right] (5) = 2 \left[ \frac{1}{2} (4)(0.15)^2 \right] \omega + 2[4(0.75 \omega)(0.75)] + \left[ \frac{1}{12} (2)(1.50)^2 \right] \omega$$

$$\omega = 0.0906 \text{ rad/s}$$

Ans: 

$$\omega = 0.0906 \text{ rad/s}$$
*19–36.

The satellite has a mass of 200 kg and a radius of gyration about z axis of \( k_z = 0.1 \) m, excluding the two solar panels \( A \) and \( B \). Each solar panel has a mass of 15 kg and can be approximated as a thin plate. If the satellite is originally spinning about the \( z \) axis at a constant rate \( \omega_z = 0.5 \) rad/s when \( \theta = 90^\circ \), determine the rate of spin if both panels are raised and reach the upward position, \( \theta = 0^\circ \), at the same instant.

**SOLUTION**

**Conservation of Angular Momentum.** When \( \theta = 90^\circ \), the mass moment of inertia of the entire satellite is

\[
I_z = 200(0.1^2) + 2 \left[ \frac{1}{12}(15)(0.3^2 + 1.5^2) + 15(0.95)^2 \right] = 34.925 \text{ kg} \cdot \text{m}^2
\]

When \( \theta = 0^\circ \), \( I'_z = 200(0.1^2) + 2 \left[ \frac{1}{12}(15)(0.3^2 + 15(0.2^2) \right] = 3.425 \text{ kg} \cdot \text{m}^2
\]

Thus

\[
(H_z)_1 = (H_z)_2
\]

\[
I_z(\omega_z)_1 = I'_z(\omega_z)_2
\]

\[
34.925(0.5) = 3.425(\omega_z)_2
\]

\[
(\omega_z)_2 = 5.0985 \text{ rad/s} = 5.10 \text{ rad/s}
\]

**Ans:**

\[
(\omega_z)_2 = 5.10 \text{ rad/s}
\]
Disk $A$ has a weight of 20 lb. An inextensible cable is attached to the 10-lb weight and wrapped around the disk. The weight is dropped 2 ft before the slack is taken up. If the impact is perfectly elastic, i.e., $e = 1$, determine the angular velocity of the disk just after impact.

**SOLUTION**

For the weight

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(2) = \frac{1}{2} \left( \frac{10}{32.2} \right) v_2^2$$

$v_2 = 11.35$ ft/s

$(H_A)_2 = (H_A)_3$

$$mv_2(0.5) + 0 = m v_3(0.5) + I_A \omega$$

$$\left( \frac{10}{32.2} \right) (11.35)(0.5) + 0 = \left( \frac{10}{32.2} \right) v_3(0.5) + \left[ \frac{1}{2} \left( \frac{20}{32.2} \right)(0.5)^2 \right] \omega$$

$$+ 1 = \frac{0.5 \omega - v_3}{v_2 - 0}$$

$$\omega = 22.7 \text{ rad/s}$$

Ans.

$$v_3 = 0$$

Ans: $\omega = 22.7 \text{ rad/s}$
The plank has a weight of 30 lb, center of gravity at G, and it rests on the two sawhorses at A and B. If the end D is raised 2 ft above the top of the sawhorses and is released from rest, determine how high end C will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about A, strikes and pivots on the sawhorses at B, and rotates clockwise off the sawhorse at A.

**SOLUTION**

Establishing a datum through AB, the angular velocity of the plank just before striking B is

\[ T_1 + V_1 = T_2 + V_2 \]

\[ 0 + 30 \left( \frac{2}{6} (1.5) \right) = \frac{1}{2} \left[ \frac{1}{12} \left( \frac{30}{32.2} \right) (9)^2 + \frac{30}{32.2} (1.5)^2 \right] (\omega_{CD})^2 + 0 \]

\( (\omega_{CD})_2 = 1.8915 \text{ rad/s} \)

\( (v_G)_2 = 1.8915 (1.5) = 2.837 \text{ m/s} \)

\( (\xi +) \quad (H_B)_2 = (H_B)_3 \)

\[ \left[ \frac{1}{12} \left( \frac{30}{32.2} \right) (9)^2 \right] (1.8915) - \frac{30}{32.2} (2.837)(1.5) = \left[ \frac{1}{2} \left( \frac{30}{32.2} \right) (9)^2 \right] (\omega_{AB})_3 + \frac{30}{32.2} (v_G)_3 (1.5) \]

Since \( (v_G)_3 = 1.5 (\omega_{AB})_3 \)

\( (\omega_{AB})_3 = 0.9458 \text{ rad/s} \)

\( (v_G)_3 = 1.4186 \text{ m/s} \)

\[ T_3 + V_3 = T_4 + V_4 \]

\[ \frac{1}{2} \left[ \frac{1}{12} \left( \frac{30}{32.2} \right) (9)^2 \right] (0.9458)^2 + \frac{1}{2} \left( \frac{30}{32.2} \right) (1.4186)^2 + 0 = 0 + 30h_G \]

\( h_G = 0.125 \)

Thus,

\[ h_C = \frac{6}{1.5} (0.125) = 0.500 \text{ ft} \]

**Ans:**

\( h_C = 0.500 \text{ ft} \)
19–39.
The 12-kg rod $AB$ is pinned to the 40-kg disk. If the disk is given an angular velocity $\omega_D = 100 \text{ rad/s}$ while the rod is held stationary, and the assembly is then released, determine the angular velocity of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing $B$. Motion is in the horizontal plane. Neglect friction at the pin $A$.

**SOLUTION**

**Conservation of Angular Momentum**

**Initial:** Since the rod is stationary and the disk is not translating, the total angular momentum about $A$ equals the angular momentum of the disk about $B$, where for the disk $I_B = \frac{1}{2} m_D r^2 = \frac{1}{2} (40)(0.3)^2 = 1.80 \text{ kg} \cdot \text{m}^2$.

$$ (H_A)_1 = I_B \omega_1 = 1.80(100) = 180 \text{ kg} \cdot \text{m}^2/\text{s} $$

**Final:** The rod and disk move as a single unit for which $I_A = \frac{1}{3} m_R l^2 + \frac{1}{2} m_D r^2 + m_D l^2$

$$ = \frac{1}{3}(12)(2)^2 + \frac{1}{2}(40)(0.3)^2 + (40)(2)^2 = 177.8 \text{ kg} \cdot \text{m}^2. $$

$$ (H_A)_2 = I_A \omega_2 = 177.8 \omega_2 $$

Setting $(H_A)_1 = (H_A)_2$ and solving,

$$ \omega_2 = 1.012 \text{ rad/s} = 1.01 \text{ rad/s} $$

**Ans:**

$$ \omega_2 = 1.01 \text{ rad/s} $$
A thin rod of mass $m$ has an angular velocity $\omega_0$ while rotating on a smooth surface. Determine its new angular velocity just after its end strikes and hooks onto the peg and the rod starts to rotate about $P$ without rebounding. Solve the problem (a) using the parameters given, (b) setting $m = 2$ kg, $\omega_0 = 4$ rad/s, $l = 1.5$ m.

**SOLUTION**

(a) 

\[
\sum (H_P)_0 = \sum (H_P)_1 \\
\left[\frac{1}{12} ml^2\right] \omega_0 = \left[\frac{1}{3} ml^2\right] \omega
\]

\[
\omega = \frac{1}{4} \omega_0 \quad \text{Ans.}
\]

(b) From part (a) \[\omega = \frac{1}{4} \omega_0 = \frac{1}{4} (4) = 1 \text{ rad/s} \quad \text{Ans.}\]
19–41.

Tests of impact on the fixed crash dummy are conducted using the 300-lb ram that is released from rest at $\theta = 30^\circ$, and allowed to fall and strike the dummy at $\theta = 90^\circ$. If the coefficient of restitution between the dummy and the ram is $e = 0.4$, determine the angle $\theta$ to which the ram will rebound before momentarily coming to rest.

**SOLUTION**

Datum through pin support at ceiling.

$T_1 + V_1 = T_2 + V_2$

$$0 - 300(\sin 30^\circ) = \frac{1}{2} \left( \frac{300}{32.2} \right) (v)^2 - 300(10)$$

$v = 17.944$ ft/s

( $\Rightarrow$ ) $e = 0.4 = \frac{v' - 0}{0 - (-17.944)}$

$v' = 7.178$ ft/s

$T_2 + V_2 = T_3 + V_3$

$$\frac{1}{2} \left( \frac{300}{32.2} \right) (7.178)^2 - 300(10) = 0 - 300(10 \sin \theta)$$

$\theta = 66.9^\circ$

Ans.
The vertical shaft is rotating with an angular velocity of 3 rad/s when $\theta = 0^\circ$. If a force $F$ is applied to the collar so that $\theta = 90^\circ$, determine the angular velocity of the shaft. Also, find the work done by force $F$. Neglect the mass of rods $GH$ and $EF$ and the collars $I$ and $J$. The rods $AB$ and $CD$ each have a mass of 10 kg.

**SOLUTION**

**Conservation of Angular Momentum:** Referring to the free-body diagram of the assembly shown in Fig. a, the sum of the angular impulses about the $z$ axis is zero. Thus, the angular momentum of the system is conserved about the axis. The mass moments of inertia of the rods about the $z$ axis when $\theta = 0^\circ$ and $90^\circ$ are

\[
(I_z)_1 = 2 \left[ \frac{1}{12} (10)(0.6^2) + 10(0.3 + 0.1)^2 \right] = 3.8 \text{ kg} \cdot \text{m}^2
\]

\[
(I_z)_2 = 2 \left[ 10(0.1^2) \right] = 0.2 \text{ kg} \cdot \text{m}^2
\]

Thus,

\[
(H_z)_1 = (H_z)_2 = 3.8(3) = 0.2 \omega_2
\]

$\omega_2 = 57 \text{ rad/s}$

**Ans.**

**Principle of Work and Energy:** As shown on the free-body diagram of the assembly, Fig. b, $W$ does negative work, while $F$ does positive work. The work of $W$ is $U_W = -Wh = -10(9.81)(0.3) = -29.43 \text{ J}$. The initial and final kinetic energy of the assembly is $T_1 = \frac{1}{2} (I_z) \omega_1^2 = \frac{1}{2} (3.8)(3^2) = 17.1 \text{ J}$ and $T_2 = \frac{1}{2} (I_z) \omega_2^2 = \frac{1}{2} (0.2)(57^2) = 324.9 \text{ J}$. Thus,

\[
T_1 + \Sigma U_{1\rightarrow 2} = T_2
\]

\[
17.1 + 2(-29.43) + U_F = 324.9
\]

$U_F = 367 \text{ J}$

**Ans.**

\[
\omega_2 = 57 \text{ rad/s}
\]

$U_F = 367 \text{ J}$
The mass center of the 3-lb ball has a velocity of \((v_G)_1 = 6 \text{ ft/s}\) when it strikes the end of the smooth 5-lb slender bar which is at rest. Determine the angular velocity of the bar about the \(z\) axis just after impact if \(e = 0.8\).

**SOLUTION**

**Conservation of Angular Momentum:** Since force \(F\) due to the impact is *internal* to the system consisting of the slender bar and the ball, it will cancel out. Thus, angular momentum is conserved about the \(z\) axis. The mass moment of inertia of the slender bar about the \(z\) axis is \(I_z = \frac{1}{12} \left( \frac{5}{32.2} \right) (4^2) = 0.2070 \text{ slug} \cdot \text{ft}^2\). Here, \(\omega_2 = \frac{(v_B)_2}{2}\).

Applying Eq. 19–17, we have

\[
(H_2)_1 = (H_2)_2
\]

\[
\left[ m_b (v_G)_1 \right] (r_b) = I_z \omega_2 + \left[ m_b (v_G)_2 \right] (r_b)
\]

\[
\left( \frac{3}{32.2} \right) (6)(2) = 0.2070 \left[ \frac{(v_B)_2}{2} \right] + \left( \frac{3}{32.2} \right) (v_G)_2 (2) \tag{1}
\]

**Coefficient of Restitution:** Applying Eq. 19–20, we have

\[
e = \frac{(v_B)_2 - (v_G)_2}{(v_G)_1 - (v_B)_1}
\]

\[
0.8 = \frac{(v_B)_2 - (v_G)_2}{6 - 0} \tag{2}
\]

Solving Eqs. (1) and (2) yields

\[(v_G)_2 = 2.143 \text{ ft/s} \quad (v_B)_2 = 6.943 \text{ ft/s}\]

Thus, the angular velocity of the slender rod is given by

\[
\omega_2 = \frac{(v_B)_2}{2} = \frac{6.943}{2} = 3.47 \text{ rad/s}\]

**Ans:**

\[
\omega_2 = 3.47 \text{ rad/s}
\]
The pendulum consists of a slender 2-kg rod \( AB \) and 5-kg disk. It is released from rest without rotating. When it falls 0.3 m, the end \( A \) strikes the hook \( S \), which provides a permanent connection. Determine the angular velocity of the pendulum after it has rotated 90°. Treat the pendulum’s weight during impact as a nonimpulsive force.

**SOLUTION**

\[
T_0 + V_0 = T_1 + V_1
\]

\[
0 + 2(9.81)(0.3) + 5(9.81)(0.3) = \frac{1}{2} (2)(v_G)_1^2 + \frac{1}{2} (5)(v_G)_1^2
\]

\((v_G)_1 = 2.4261 \text{ m/s}\)

\[
\Sigma (H_1) = \Sigma (H_2)
\]

\[
2(2.4261)(0.25) + 5(2.4261)(0.7) = \left[ \frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2 \right] \omega
\]

\(\omega = 3.572 \text{ rad/s}\)

\[
T_2 + V_2 = T_3 + V_3
\]

\[
\frac{1}{2} \left[ \frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2 \right] (3.572)^2 + 0
\]

\[
= \frac{1}{2} \left[ \frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2 \right] \omega^2
\]

\[+ 2(9.81)(0.25) + 5(9.81)(0.7)\]

\(\omega = 6.45 \text{ rad/s}\)

**Ans:**

\(\omega = 6.45 \text{ rad/s}\)
19–45.

The 10-lb block slides on the smooth surface when the corner $D$ hits a stop block $S$. Determine the minimum velocity $v$ the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of $S$. *Hint:* During impact consider the weight of the block to be nonimpulsive.

**SOLUTION**

**Conservation of Energy:** If the block tips over about point $D$, it must at least achieve the dash position shown. Datum is set at point $D$. When the block is at its initial and final position, its center of gravity is located 0.5 ft and 0.7071 ft above the datum. Its initial and final potential energy are $10(0.5) = 5.00 \text{ ft} \cdot \text{lb}$ and $10(0.7071) = 7.071 \text{ ft} \cdot \text{lb}$. The mass moment of inertia of the block about point $D$ is

$$I_D = \frac{1}{12} \left( \frac{10}{32.2} \right) (1^2 + 1^2) + \left( \frac{10}{32.2} \right) (\sqrt{0.5^2 + 0.5^2})^2 = 0.2070 \text{ slug} \cdot \text{ft}^2$$

The initial kinetic energy of the block (after the impact) is $\frac{1}{2} I_D \omega_2^2 = \frac{1}{2} (0.2070) \omega_2^2$. Applying Eq. 18–18, we have

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2} (0.2070) \omega_2^2 + 5.00 = 0 + 7.071$$

$$\omega_2 = 4.472 \text{ rad/s}$$

**Conservation of Angular Momentum:** Since the weight of the block and the normal reaction $N$ are nonimpulsive forces, the angular momentum is conserved about point $D$. Applying Eq. 19–17, we have

$$(H_D)_1 = (H_D)_2$$

$$(mv_G)(r') = I_D \omega_2$$

$$\left[ \left( \frac{10}{32.2} \right) v \right] (0.5) = 0.2070(4.472)$$

$$v = 5.96 \text{ ft/s}$$

*Ans.*

$$v = 5.96 \text{ ft/s}$$
19–46.

Determine the height $h$ at which a billiard ball of mass $m$ must be struck so that no frictional force develops between it and the table at $A$. Assume that the cue $C$ only exerts a horizontal force $P$ on the ball.

**SOLUTION**

For the ball

$$\left( \frac{\dot{v}}{v} \right) mv_1 + \sum F dt = mv_2$$

$$0 + P(\Delta t) = mv_2$$

$$\zeta + (H_A)_1 + \sum M_{A, \Delta t} = (H_A)_2$$

$$0 + (P)\Delta t(h) = \left[ \frac{2}{5} m r^2 + m r^2 \right]v_2$$

Require $v_2 = \omega_2 r$

Solving Eqs. (1)–(3) for $h$ yields

$$h = \frac{7}{5} r$$

**Ans.**
19–47.

The pendulum consists of a 15-kg solid ball and 6-kg rod. If it is released from rest when \( \theta_1 = 90^\circ \), determine the angle \( \theta_2 \) after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take \( e = 0.6 \).

**SOLUTION**

**Kinetic Energy.** The mass moment of inertia of the pendulum about \( A \) is
\[
I_A = \frac{1}{3}(6)^2 + \left[ \frac{2}{5}(15)(0.3)^2 + 15(2.3)^2 \right] = 87.89 \text{ kg} \cdot \text{m}^2.
\]
Thus,
\[
T = \frac{1}{2}I_A \omega^2 = \frac{1}{2} (87.89) \omega^2 = 43.945 \omega^2.
\]

**Potential Energy.** With reference to datum set in Fig. 1, the gravitational potential energy of the pendulum is
\[
V_g = m g y_r + m g y_s = 6(9.81)(- \cos \theta) + 15(9.81)(-2.3 \cos \theta) = -397.305 \cos \theta.
\]

**Coefficient of Restitution.** The velocity of the mass center of the ball is
\[
v_b = \omega G = \omega (2.3).
\]
Thus
\[
\epsilon = \frac{(v_w)_2 - (v_b)_2}{(v_b)_2 - (v_w)_1} = \frac{0 - [-\omega^2(2.3)]}{\omega^2(2.3) - 0} = 0.6 \omega_2^2.
\]

**Conservation of Energy.** Consider the pendulum swing from the position \( \theta = 90^\circ \) to \( \theta = 0^\circ \) just before the impact,
\[
(V_g)_1 = -397.305 \cos 90^\circ = 0
\]
\[
(V_g)_2 = -397.305 \cos 0^\circ = -397.305 \text{ J}
\]
\[
T_1 = 0 \quad T_2 = 43.945 \omega_2^2
\]

Then
\[
T_1 + V_1 = T_2 + V_2
\]
\[
0 + 0 = 43.945 \omega_2^2 + (-397.305)
\]
\[
\omega_2 = 3.0068 \text{ rad/s}
\]
Thus, just after the impact, from Eq. (1)
\[
\omega_2 = 0.6(3.0068) = 1.8041 \text{ rad/s}
\]
Consider the pendulum swing from position \( \theta = 90^\circ \) just after the impact to \( \theta \),
\[
(V_g)_2' = (V_g)_2 = -397.305 \text{ J}
\]
\[
(V_g)_3 = -397.305 \cos \theta
\]
\[
T_2' = 43.945(1.8041^2) = 143.03 \text{ J}
\]
\[
T_3 = 0 \quad \text{ (required)}
\]

Then
\[
T_2' + V_2' = T_3 + V_3
\]
\[
143.03 + (-397.305) = 0 + (-397.305 \cos \theta)
\]
\[
\theta = 50.21^\circ = 50.2^\circ
\]

Ans.

\[\theta = 50.2^\circ\]
The 4-lb rod $AB$ is hanging in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end $B$. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at $B$ is $e = 0.8$.

SOLUTION

Conservation of Angular Momentum: Since force $F$ due to the impact is internal to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point $A$. The mass moment of inertia of the slender rod about point $A$ is $I_A = \frac{1}{12} \left( \frac{4}{32.2} \right) (3^2) + \frac{4}{32.2} (1.5^2) = 0.3727 \text{ slug} \cdot \text{ft}^2$.

Here, $\omega_2 = \frac{(v_B)_2}{3}$. Applying Eq. 19–17, we have

$$ (H_A)_1 = (H_A)_2 $$

$$ [m_b(v_b)_1](r_b) = I_A \omega_2 + [m_b(v_b)_2](r_b) $$

$$ \left( \frac{2}{32.2} \right) (12)(3) = 0.3727 \left( \frac{(v_B)_2}{3} \right) + \left( \frac{2}{32.2} \right) (v_b)_2 (3) \quad [1] $$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$ e = \frac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_b)_1} $$

$$ (\therefore) \quad 0.8 = \frac{(v_B)_2 - (v_b)_2}{12 - 0} \quad [2] $$

Solving Eqs. [1] and [2] yields

$$ (v_b)_2 = 3.36 \text{ ft/s} \rightarrow \quad \text{Ans.} $$

$$ (v_B)_2 = 12.96 \text{ ft/s} \rightarrow $$

Ans:

$$ (v_b)_2 = 3.36 \text{ ft/s} \rightarrow $$
19–49.

The hammer consists of a 10-kg solid cylinder \( C \) and a 6-kg uniform slender rod \( AB \). If the hammer is released from rest when \( \theta = 90^\circ \) and strikes the 30-kg block \( D \) when \( \theta = 0^\circ \), determine the velocity of block \( D \) and the angular velocity of the hammer immediately after the impact. The coefficient of restitution between the hammer and the block is \( e = 0.6 \).

**SOLUTION**

**Conservation of Energy:** With reference to the datum in Fig. a, \( V_1 = (V_G)_1 = W_{AB}(y_{GAB})_1 + W_C(y_{GC})_1 = 0 \) and \( V_2 = (V_G)_2 = -W_{AB}(y_{GAB})_2 - W_C(y_{GC})_2 = -6(9.81)(0.25) - 10(9.81)(0.55) = -68.67 \) J. Initially, \( T_1 = 0 \). Since the hammer rotates about the fixed axis, \( (v_{GAB})_2 = \omega_{GAB} = \omega_2(0.25) \) and \( (v_{GC})_2 = \omega_2 r_{GC} = \omega_2(0.55) \). The mass moment of inertia of rod \( AB \) and cylinder \( C \) about their mass centers is \( I_{GAB} = \frac{1}{12} ml^2 = \frac{1}{12} (6)(0.5^2) = 0.125 \text{ kg} \cdot \text{m}^2 \) and \( I_C = \frac{1}{12} m(3r^2 + h^2) = \frac{1}{12} (10)
\left[ 3(0.05)^2 + 0.15^2 \right] = 0.025 \text{ kg} \cdot \text{m}^2 \). Thus,

\[
T_2 = \frac{1}{2} I_{GAB} \omega_2^2 + \frac{1}{2} m_{AB} (v_{GAB})_2^2 + \frac{1}{2} I_C \omega_2^2 + \frac{1}{2} m_C (v_{GC})_2^2
= \frac{1}{2} (0.125) \omega_2^2 + \frac{1}{2} (6) [\omega_2(0.25)]^2 + \frac{1}{2} (0.025) \omega_2^2 + \frac{1}{2} (10) [\omega_2(0.55)]^2
= 1.775 \omega_2^2
\]

Then,

\[
T_1 + V_1 = T_2 + V_2
\]

\[
0 + 0 = 1.775 \omega_2^2 + (-68.67)
\]

\[
\omega_2 = 6.220 \text{ rad/s}
\]

**Conservation of Angular Momentum:** The angular momentum of the system is conserved point \( A \). Then,

\[
(H_A)_1 = (H_A)_2
\]

\[
0.125(6.220) + 6[6.220(0.25)](0.25) + 0.025(6.220) + 10[6.220(0.55)](0.55)
= 30v_D(0.55) - 0.125\omega_3 - 6[\omega_3(0.25)](0.25) - 0.025\omega_3 - 10[\omega_3(0.55)](0.55)
\]

\[
16.5v_D - 3.55\omega_3 = 22.08
\]
19–49. Continued

**Coefficient of Restitution:** Referring to Fig. c, the components of the velocity of the impact point \( P \) just before and just after impact along the line of impact are 
\[
(v_P)_x^2 = (v_{GC})_2 = \omega_2 \omega_{GC} = 6.220(0.55) = 3.421 \text{ m/s} \quad \text{and} \quad (v_P)_x^3 = (v_{GC})_3 = \omega_3 \omega_{GC} = \omega_3 (0.55) \leftarrow .
\]
Thus,
\[
\begin{align*}
\dot{e} &= \frac{(v_D)_3 - [(v_P)_x^3]}{[(v_P)_x^2] - (v_D)_2} \\
0.6 &= \frac{(v_D)_3 - [\omega_3 (0.55)]}{3.421 - 0} \\
(v_D)_3 + 0.55 \omega_3 &= 2.053 \quad (2)
\end{align*}
\]
Solving Eqs. (1) and (2),
\[
(v_D)_3 = 1.54 \text{ m/s} \quad \omega_3 = 0.934 \text{ rad/s} \quad \text{Ans.}
\]
The 20-kg disk strikes the step without rebounding. Determine the largest angular velocity $\omega_1$ the disk can have and not lose contact with the step, $A$.

**SOLUTION**

**Conservation of Angular Momentum.** The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.2^2) = 0.4 \text{ kg} \cdot \text{m}^2$. Since no slipping occurs, $v_G = \omega r = \omega (0.2)$. Referring to the impulse and momentum diagram, Fig. a, we notice that angular moment is conserved about point $A$ since $\mathbf{W}$ is nonimpulsive. Thus,

$$I_G |_{t_1} = I_G |_{t_2}$$

$$20[\omega_1(0.2)](0.17) + 0.4 \omega_1 = 0.4 \omega_2 + 20[\omega_2(0.2)](0.2)$$

$$\omega_1 = 1.1111 \omega_2 \quad (1)$$

**Equations of Motion.** Since the requirement is the disk is about to lose contact with the step when it rotates about $A$, $N_A = 0$. Here $\theta = \cos^{-1}\left(\frac{0.17}{0.2}\right) = 31.79^\circ$. Consider the motion along $n$ direction,

$$\sum F_n = M(a_G)_n; \quad 20(9.81) \cos 31.79^\circ = 20[\omega_2^2(0.2)]$$

$$\omega_2 = 6.4570 \text{ rad/s}$$

Substitute this result into Eq. (1)

$$\omega_1 = 1.1111(6.4570) = 7.1744 \text{ rad/s} = 7.17 \text{ rad/s} \quad \text{Ans.}$$
19–51.

The solid ball of mass $m$ is dropped with a velocity $v_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $v_2$, determine the angle $\theta$ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is $e$.

**SOLUTION**

**Conservation of Angular Momentum:** Since the weight of the solid ball is a nonimpulsive force, then angular momentum is conserved about point $A$. The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$. Here, $\omega_2 = \frac{v_2 \cos \theta}{r}$. Applying Eq. 19–17, we have

$$\begin{align*}
(H_A)_1 &= (H_A)_2 \\
\left[ m_b (v_b)_1 \right] (r') &= I_G \omega_2 + \left[ m_b (v_b)_2 \right] (r^*) \\
(mv_1)(r \sin \theta) &= \left( \frac{2}{5}mr^2 \right) \left( \frac{v_1 \cos \theta}{r} \right) + (mv_2)(r \cos \theta) \\
\frac{v_2}{v_1} &= \frac{5}{7} \tan \theta
\end{align*}$$

(1)

**Coefficient of Restitution:** Applying Eq. 19–20, we have

$$e = \frac{(v_b)_2 - (v_b)_1}{\left| (v_b)_1 - 0 \right|} = -\frac{(v_2 \sin \theta)}{-v_1 \cos \theta}$$

$$v_2 = e \frac{\cos \theta}{\sin \theta}$$

Equating Eqs. (1) and (2) yields

$$\frac{5}{7} \tan \theta = e \frac{\cos \theta}{\sin \theta}$$

$$\tan^2 \theta = \frac{7}{5}e$$

$$\theta = \tan^{-1} \left( \frac{\sqrt{7} e}{\sqrt{5}} \right)$$

Ans.
*19–52.

The wheel has a mass of 50 kg and a radius of gyration of 125 mm about its center of mass \( G \). Determine the minimum value of the angular velocity \( \omega_1 \) of the wheel, so that it strikes the step at \( A \) without rebounding and then rolls over it without slipping.

**SOLUTION**

**Conservation of Angular Momentum:** Referring to Fig. \( a \), the sum of the angular impulses about point \( A \) is zero. Thus, angular momentum of the wheel is conserved about this point. Since the wheel rolls without slipping, \( (v_G)_1 = \omega_1 r = \omega_1 (0.15) \) and \( (v_G)_2 = \omega_2 r = \omega_2 (0.15) \). The mass moment of inertia of the wheel about its mass center is \( I_G = mk_G^2 = 50(0.125^2) = 0.78125 \) kg \( \cdot \) m\(^2\). Thus,

\[
(H_A)_1 = (H_A)_2

50[\omega_1(0.15)](0.125) + 0.78125\omega_1 = 50[\omega_2(0.15)](0.15) + 0.78125\omega_2

\omega_1 = 1.109\omega_2 \tag{1}
\]

**Conservation of Energy:** With reference to the datum in Fig. \( a \), \( V_2 = (V_G)_2 = W(y_G)_2 = 0 \) and \( V_3 = (V_G)_3 = W(y_G)_3 = 50(9.81)(0.025) = 12.2625 \) J. Since the wheel is required to be at rest in the final position, \( T_3 = 0 \). The initial kinetic energy of the wheel is \( T_2 = \frac{1}{2}m(v_G)_1^2 + \frac{1}{2}I_G\omega_1^2 = \frac{1}{2}(50)[\omega_1(0.15)]^2 + \frac{1}{2}(0.78125)(\omega_1^2) = 0.953125\omega_1^2 \). Then

\[
T_2 + V_2 = T_3 + V_3

0.953125\omega_1^2 + 0 = 0 + 12.2625

\omega_1 = 3.98 \text{ rad/s}
\]

Substituting this result into Eq. (1), we obtain

\[
\omega_1 = 3.98 \text{ rad/s}
\]

**Ans.**
The wheel has a mass of 50 kg and a radius of gyration of 125 mm about its center of mass $G$. If it rolls without slipping with an angular velocity of $\omega_1 = 5 \text{ rad/s}$ before it strikes the step at $A$, determine its angular velocity after it rolls over the step. The wheel does not lose contact with the step when it strikes it.

**SOLUTION**

**Conservation of Angular Momentum:** Referring to Fig. (a), the sum of the angular impulses about point $A$ is zero. Thus, angular momentum of the wheel is conserved about this point. Since the wheel rolls without slipping, $(v_G)_1 = \omega_1 r = (5)(0.15) = 0.75 \text{ m/s}$ and $\omega_2 = \omega_G = \omega_2(0.15)$. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G^2 = 50(0.125^2) = 0.78125 \text{ kg} \cdot \text{m}^2$. Thus,

$$\begin{align*}
(H_A)_1 &= (H_A)_2 \\
50(0.75)(0.125) + 0.78125(5) &= 50[\omega_2(0.15)](0.15) + 0.78125\omega_2 \\
\omega_2 &= 4.508 \text{ rad/s} \tag{1}
\end{align*}$$

**Conservation of Energy:** With reference to the datum in Fig. (a), $V_2 = (V_G)_2 = W(y_G)_2 = 0$ and $V_3 = (V_G)_3 = W(y_G)_3 = 50(9.81)(0.025) = 12.2625 \text{ J}$. The initial kinetic energy of the wheel is $T_1 = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(50)[(0.75)^2 + \frac{1}{2}(0.78125)\omega^2 = 0.953125\omega^2$. Thus, $T_2 = 0.953125\omega^2 = 0.953125(4.508^2) = 19.37 \text{ J}$ and $T_3 = 0.953125(2.73^2)$. 

$$\begin{align*}
T_2 + V_2 &= T_3 + V_3 \\
19.37 + 0 &= 0.953125\omega_3^2 + 12.2625 \\
\omega_3 &= 2.73 \text{ rad/s} \quad \text{Ans.}
\end{align*}$$
The rod of mass $m$ and length $L$ is released from rest without rotating. When it falls a distance $L$, the end $A$ strikes the hook $S$, which provides a permanent connection. Determine the angular velocity $\omega$ of the rod after it has rotated $90^\circ$. Treat the rod’s weight during impact as a nonimpulsive force.

**SOLUTION**

\[ T_1 + V_1 = T_2 + V_2 \]

\[ 0 + mgL = \frac{1}{2}mv_G^2 + 0 \]

\[ v_G = \sqrt{2gL} \]

\[ H_1 = H_2 \]

\[ m\sqrt{2gL\left(\frac{L}{2}\right)} = \frac{1}{3}mL^2(\omega_2) \]

\[ \omega_2 = \frac{3\sqrt{2gL}}{L} \]

\[ T_2 + V_2 = T_3 + V_3 \]

\[ \frac{1}{2}\left(\frac{1}{3}mL^2\right)\frac{9(2gL)}{4L^2} + 0 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 - mg\left(\frac{L}{2}\right) \]

\[ \frac{3}{4}gL = \frac{1}{6}L^2\omega^2 - g\left(\frac{L}{2}\right) \]

\[ \omega = \sqrt{\frac{7.5gL}{L}} \]

**Ans:**

\[ \omega = \sqrt{\frac{7.5gL}{L}} \]
19–55.

The 15-lb rod $AB$ is released from rest in the vertical position. If the coefficient of restitution between the floor and the cushion at $B$ is $e = 0.7$, determine how high the end of the rod rebounds after impact with the floor.

**SOLUTION**

$T_1 = V_1 = T_2 + V_2$

$$0 + 15(1) = \frac{1}{2} \left[ \frac{1}{3} \left( \frac{15}{32.2} \right) (2)^2 \right] \omega_2^2$$

$\omega_2 = 6.950 \text{ rad/s}$  
Hence $(v_B)_3 = 6.950(2) = 13.90 \text{ rad/s}$

$$\left( + \right) e = \frac{0 - (v_B)_3}{(v_B)_2 - 0}; \quad 0.7 = \frac{0 - (v_B)_3}{13.90}$$

$(v_B)_3 = -9.730 \text{ ft/s} = 9.730 \text{ ft/s}$  

$$\omega_3 = \frac{(v_B)_3}{2} = \frac{9.730}{2} = 4.865 \text{ rad/s}$$

$T_3 + V_3 = T_4 + V_4$

$$\frac{1}{2} \left[ \frac{1}{3} \left( \frac{15}{32.2} \right) (2)^2 \right] (4.865)^2 = 0 + 15(h_G)$$

$h_G = 0.490 \text{ ft}$

$h_B = 2h_G = 0.980 \text{ ft}$

Ans.

**Ans:**

$h_B = 0.980 \text{ ft}$
A ball having a mass of 8 kg and initial speed of \(v_1 = 0.2 \text{ m/s}\) rolls over a 30-mm-long depression. Assuming that the ball rolls off the edges of contact first \(A\), then \(B\), without slipping, determine its final velocity \(v_2\) when it reaches the other side.

**SOLUTION**

\[
\omega_1 = \frac{0.2}{0.125} = 1.6 \text{ rad/s} \quad \omega_2 = \frac{v_2}{0.125} = 8v_2
\]

\[
\theta = \sin^{-1} \left( \frac{15}{125} \right) = 6.8921^\circ
\]

\[
h = 125 \cos 6.8921^\circ = 0.90326 \text{ mm}
\]

\[
T_1 + V_1 = T_2 + V_2
\]

\[
\frac{1}{2} (8) (0.2)^2 + \frac{1}{2} \left[ \frac{2}{5} (8)(0.125)^2 \right](1.6)^2 = 0
\]

\[
= -(0.90326)(10^{-3})8(9.81) + \frac{1}{2} (8)\omega_2^2 (0.125)^2 + \frac{1}{2} \left[ \frac{2}{5} (8)(0.125)^2 \right](\omega)^2
\]

\[
\omega = 1.836 \text{ rad/s}
\]

\[
(H_B)_2 = (H_B)_3
\]

\[
\frac{2}{5} (8)(0.125)^2 \left[ (1.836) + 8(1.836)(0.125) \cos 6.892^\circ (0.125 \cos 6.892^\circ) \right]
\]

\[
- 8(0.22948 \sin 6.892^\circ)(0.125 \sin 6.892^\circ)
\]

\[
= \left[ \frac{2}{5} (8)(0.125)^2 \right] \omega_3 + 8(0.125)\omega_3 (0.125)
\]

\[
\omega_3 = 1.7980 \text{ rad/s}
\]

\[
T_3 + V_3 = T_4 + V_4
\]

\[
\frac{1}{2} \left[ \frac{2}{5} (8)(0.125)^2 \right] (1.7980)^2 + \frac{1}{2} (8)(1.7980)(0.125)^2 + 0
\]

\[
= 8(9.81)(0.90326(10^{-3})) + \frac{1}{2} \left[ \frac{2}{5} (8)(0.125)^2 \right](\omega_4)^2
\]

\[
\omega_4 = 1.56 \text{ rad/s}
\]

So that

\[
v_2 = 1.56(0.125) = 0.195 \text{ m/s}
\]

**Ans.**

\[
v_2 = 0.195 \text{ m/s}
\]
19–57.
A solid ball with a mass \( m \) is thrown on the ground such that at the instant of contact it has an angular velocity \( \omega_1 \) and velocity components \( (v_G)_x \) and \( (v_G)_y \) as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is \( e \).

**SOLUTION**

Coefficient of Restitution (y direction):

\[
\begin{align*}
(+) & \quad e = \frac{0 - (v_G)_y^2}{(v_G)_y} = -e(v_G)_y = e(v_G)_y \\
& \quad \text{Ans.}
\end{align*}
\]

Conservation of angular momentum about point on the ground:

\[
(+) \quad (H_A)_1 = (H_A)_2
\]

\[
-\frac{2}{5}mr^2\omega_1 + m(v_G)_x r = \frac{2}{5}mr^2\omega_2 + m(v_G)_x^2 r
\]

Since no slipping, \( (v_G)_x = \omega_2 r \) then,

\[
\omega_2 = \frac{5\left((v_G)_x - \frac{2}{5}\omega r\right)}{7r}
\]

Therefore

\[
(v_G)_x = \frac{5}{7}\left((v_G)_x - \frac{2}{5}\omega_1 r\right)
\]

**Ans:**

\[
\begin{align*}
(v_G)_x &= e(v_G)_x \\
(v_G)_x &= \frac{5}{7}\left((v_G)_x - \frac{2}{5}\omega_1 r\right)
\end{align*}
\]
19–58.

The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when \( \theta_0 = 0^\circ \), determine the angle \( \theta_1 \) of rebound after the ball strikes the wall and the pendulum swings up to the point of momentary rest. Take \( e = 0.6 \).

**SOLUTION**

\[
I_A = \frac{1}{3} \left( \frac{4}{32.2} \right) (2)^2 + \frac{2}{5} \left( \frac{10}{32.2} \right) (0.3)^2 + \left( \frac{10}{32.2} \right) (2.3)^2 = 1.8197 \text{ slug} \cdot \text{ft}^2
\]

Just before impact:

\[
T_1 + V_1 = T_2 + V_2
\]

\[
0 + 0 + \frac{1}{2} (1.8197) \omega^2 - 4(1) - 10(2.3)
\]

\[\omega = 5.4475 \text{ rad/s}\]

\[v_p = 2.3(5.4475) = 12.529 \text{ ft/s}\]

Since the wall does not move,

\[
\left( \begin{array}{c} \downarrow \varepsilon \end{array} \right) e = 0.6 = \frac{(v_p) - 0}{0 - (-12.529)}
\]

\[v_p = 7.518 \text{ ft/s}\]

\[\omega' = \frac{7.518}{2.3} = 3.2685 \text{ rad/s}\]

\[T_3 + V_3 = T_4 + V_4\]

\[
\frac{1}{2} (1.8197)(3.2685)^2 - 4(1) - 10(2.3) = 0 - 4(1) \sin \theta_1 - 10(2.3 \sin \theta_1)
\]

\[\theta_1 = 39.8^\circ\]

**Ans:**

\[\theta_1 = 39.8^\circ\]