CHAPTER 10

Decision Making Under Uncertainty

DECISION AND RISK ANALYSIS AT DU PONT

Formal decision analysis in the face of uncertainty frequently occurs at the most strategic levels of a company's planning process and typically involves teams of high-level managers from all areas of the company. This is certainly the case with Du Pont, as reported by two internal decision analysis experts, Krumm and Rolle (1992), in their article "Management and Application of Decision and Risk Analysis in Du Pont." Du Pont's formal use of decision analysis began in the 1960s, but because of a lack of computing power and distrust of the method by senior-level management, it never really got a foothold. However, by the mid-1980s things had changed considerably. The company was involved in a faster-moving, more uncertain environment, more people throughout the company were empowered to make decisions, and these decisions had to be made more quickly. In addition, the computing power had arrived to make large-scale quantitative analysis feasible. Since that time, Du Pont has embraced formal decision-making analysis in all its businesses, and the trend is almost certain to continue.

The article describes a typical example of decision analysis within the company. One of Du Pont's businesses, Business Z (so-called for reasons of confidentiality), was stagnating. It was not set up to respond quickly to changing customer demands, and its financial position was declining due to lower prices and market share. A decision board and a project team were empowered to turn things around. The project team developed a detailed timetable to accomplish three basic steps: frame the problem, assess uncertainties and perform the analysis, and implement the recommended decision. The first step involved setting up a "strategy table" to list the possible strategies and the factors that would affect or be affected by them. The three basic strategies were (1) a base-case strategy (continue operating as is), (2) a product differentiation strategy (develop new products), and (3) a cost leadership strategy (shut down the plant and streamline the product line).

In the second step, the team asked a variety of experts throughout the company for their assessments of the likelihood of key uncertain events. In the analysis step they then used all of the information gained to determine the strategy with the largest expected net present value. Two important aspects of this analysis step were the extensive use of sensitivity analysis (many what-if questions) and the emergence of new "hybrid" strategies that dominated the strategies that had been considered to that point. In particular, the team finally decided on a product differentiation strategy that also decreased costs by shutting down some facilities in each plant.

By the time of the third step, implementation, the decision board needed little convincing. Since all of the key people had been given the opportunity to provide input to the process, everyone was convinced that the right strategy had been selected. All that was left was to put the plan in motion and monitor its results. The results were impressive. Business Z made a complete turnaround, and its net present value increased by close to $200 million. Besides this tangible benefit, there were definite intangible benefits from the overall process. As Du Pont's vice president for finance said, "The D&RA [decision and risk analysis] process improved communication within the business team as well as between the team and corporate management, resulting in rapid approval and execution. As a decision maker, I highly value such a clear and logical approach to making choices under uncertainty and will continue to use D&RA whenever possible." ■

10.1 INTRODUCTION

In this chapter we will provide a formal framework for analyzing decision problems that involve uncertainty. We will discuss the most frequently used criteria for choosing among alternative decisions, how probabilities are used in the decision-making process, how decisions made at an early stage affect decisions made at a later stage, how a decision maker can quantify the value of information, and how attitudes toward risk can affect the analysis. Throughout, we will employ a powerful graphical tool—decision trees—to guide the analysis. A decision tree enables the decision maker to view all important aspects of the problem at once: the decision alternatives, the uncertain outcomes and their probabilities, the economic consequences, and the chronological order of events. We will show how to implement decision trees in Excel by taking advantage of a very powerful and flexible add-in from Palisade called PrecisionTree.

Many examples of decision making under uncertainty exist in the business world. Here are several examples.

- Companies routinely place bids for contracts to complete a certain project within a fixed time frame. Often these are sealed bids, where each of several companies presents in a sealed envelope a bid for completing the project; then the envelopes are opened, and the low bidder is awarded the bid amount to complete the project. Any particular company in the bidding competition must deal with the possible uncertainty of its *actual* cost of completing the project (should it win the bid), as well as the uncertainty involved in what the other companies will bid. The trade-off is between bidding low in order to win the bid and bidding high in order to make a profit.
- Whenever a company contemplates introducing a new product into the market, there are a number of uncertainties that affect the decision, probably the most important being the customers' reaction to this product. If the product generates high customer demand, then the company will make a large profit. But if demand is low (and, after all, the vast majority of new products do poorly), then the company might not even recoup its development costs. Because the level of customer demand is critical, the company might try to gauge this level by test marketing the product in one region of the country. If this test market is a success, the company can then be more optimistic that a full-scale national marketing of the product will also be successful. But if the test market is a failure, the company can cut its losses by abandoning the product.
- Borison (1995) describes an application of formal decision analysis by Oglethorpe Power Corporation (OPC), a Georgia-based electricity supplier. The basic decision OPC faced was whether to build a new transmission line to supply large amounts of electricity to parts of Florida and, if they decided to build it, how to finance this project. OPC had to deal with several sources of uncertainty: the cost of building new facilities, the demand for power in Florida, and various market conditions, such as the spot price of electricity.
- Ulvila (1987) describes the decision analysis performed by the U.S. Postal Service regarding the purchase of automation equipment. One of the investment decisions was which type of OCR (optical character recognition) equipment the Postal Service should purchase (or convert) for reading single- and/or multiple-line addresses on packages. An important factor in this decision was the level of use by businesses of the "zip+4" (nine-digit zip codes). Zip+4 usage had been recommended for some time but was used only sporadically. The Postal Service was uncertain about the future level of business zip+4 usage. If businesses used the nine-digit codes heavily in the future, then a certain type of (expensive) OCR equipment would be most economical. If business use of zip+4 did not increase, then purchasing this equipment would be a waste of money. The decision was an extremely important one, given the expense of the proposed equipment and the fact that the Postal Service would have to live with whatever equipment it purchased for a number of years.
- Utility companies must make many decisions that have significant environmental and economic consequences. [Balson et al. (1992) provide a good discussion of such consequences.] For these companies it is not necessarily enough to conform to federal or state environmental regulations. Recent court decisions have found companies liable—for huge settlements—when accidents occurred, even though the companies followed all existing regulations. Therefore, when utility companies decide, say, whether to replace equipment or mitigate the effects of environmental

pollution, they must take into account the possible environmental consequences (such as injuries to people) as well as economic consequences (such as lawsuits). An aspect of these situations that makes decision analysis particularly difficult is that the potential "disasters" are often extremely improbable; hence, their likelihoods are very difficult to assess accurately.

10.2 ELEMENTS OF A DECISION ANALYSIS

Although decision making under uncertainty occurs in a wide variety of contexts, all problems have three elements in common: (1) the set of decisions (or strategies) available to the decision maker, (2) the set of possible outcomes and the probabilities of these outcomes, and (3) a value model that prescribes results, usually monetary values, for the various combinations of decisions and outcomes. Once these elements are known, the decision maker can find an "optimal" decision, depending on the optimality criterion chosen. Rather than discussing these elements in the abstract, we introduce them in the context of the following extended example.

EXAMPLE 10.1

BIDDING FOR A GOVERNMENT CONTRACT AT SCITOOLS

SciTools Incorporated, a company that specializes in scientific instruments, has been invited to make a bid on a government contract. The contract calls for a specific number of these instruments to be delivered during the coming year. The bids must be sealed (so that no company knows what the others are bidding), and the low bid wins the contract. SciTools estimates that it will cost \$5000 to prepare a bid and \$95,000 to supply the instruments if it wins the contract. On the basis of past contracts of this type, SciTools believes that the possible low bids from the competition, if there is any competition, and the associated probabilities are those shown in Table 10.1. In addition, SciTools believes there is a 30% chance that there will be *no* competing bids.

TABLE 10.1 Data for Bidding Example

Low Bid	Probability
Less than \$115,000	0.2
Between \$115,000 and \$120,000	0.4
Between \$120,000 and \$125,000	0.3
Greater than \$125,000	0.1

Solution

Let's discuss the three elements of SciTools' problem. First, SciTools has two basic strategies: submit a bid or do not submit a bid. If SciTools submits a bid, then it must decide how much to bid. Based on SciTools' cost to prepare the bid and its cost to supply the instruments, there is obviously no point in bidding less than \$100,000—SciTools wouldn't make a profit even if it won the bid. Although any bid amount over

$100,000 might be considered, the data in Table 10.1 might persuade SciTools to limit its choices to $115,000, $120,000, and $125,000.[1]

The next element of the problem involves the uncertain outcomes and their probabilities. We have assumed that SciTools knows exactly how much it will cost to prepare a bid and how much it will cost to supply the instruments if it wins the bid. (In reality, these are probably estimates of the actual costs.) Therefore, the only source of uncertainty is the behavior of the competitors—will they bid, and if so, how much? From SciTools' standpoint, this is difficult information to obtain. The behavior of the competitors depends on (1) how many competitors are likely to bid and (2) how the competitors assess *their* costs of supplying the instruments. However, we will assume that SciTools has been involved in similar bidding contests in the past and can, therefore, predict competitor behavior from past competitor behavior. The result of such prediction is the assessed probability distribution in Table 10.1 and the 30% estimate of the probability of no competing bids.

The last element of the problem is the value model that transforms decisions and outcomes into monetary values for SciTools. The value model is straightforward in this example, but it can become quite complex in other applications, especially when the time value of money is involved and some quantities (such as the costs of environmental pollution) are difficult to quantify. If SciTools decides right now not to bid, then its monetary value is $0—no gain, no loss. If it makes a bid and is underbid by a competitor, then it loses $5000, the cost of preparing the bid. If it bids B dollars and wins the contract, then it makes a profit of $B - \$100{,}000$, that is, B dollars for winning the bid, less $5000 for preparing the bid, less $95,000 for supplying the instruments. For example, if it bids $115,000 and the lowest competing bid, if any, is greater than $115,000, then SciTools makes a profit of $15,000.

It is often convenient to list the monetary outcomes in a **payoff table**, as shown in Table 10.2. For each possible decision and each possible outcome, the payoff table lists the monetary value to SciTools, where a positive value represents a profit and a negative value represents a loss. At the bottom of the table, we list the probabilities of the various outcomes. For example, the probability that the competitors' low bid is less than $115,000 is 0.7 (the probability of at least one competing bid) multiplied by 0.2 (the probability that the lowest competing bid is less than $115,000, given that there is at least one competing bid).

It is sometimes possible to simplify payoff tables to better understand the essence of the problem. In the present example, if SciTools bids, then the only necessary information about the competitors' bid is whether it is lower or higher than SciTools'

TABLE 10.2 Payoff Table for SciTools Bidding Example

		Competitors' Low Bid ($1000s)				
		No Bid	**<115**	**>115, <120**	**>120, <125**	**>125**
SciTools'	**No Bid**	0	0	0	0	0
Bid	**115**	15	−5	15	15	15
($1000s)	**120**	20	−5	−5	20	20
	125	25	−5	−5	−5	25
Probability		0.3	0.7(0.2)	0.7(0.4)	0.7(0.3)	0.7(0.1)

[1]The problem with a bid such as $117,000 is that the data in Table 10.1 make it impossible to calculate the probability of SciTools winning the contract if it bids this amount. Other than this, however, there is nothing that rules out such an "in-between" bid.

bid. That is, SciTools cares only whether it wins the contract or not. Therefore, an alternative way of presenting the payoff table is shown in Table 10.3.

The third and fourth columns of this table indicate the payoffs to SciTools, depending on whether it wins or loses the bid. The rightmost column shows the probability that SciTools wins the bid for each possible decision. For example, if SciTools bids \$120,000, then it wins the bid if there are no competing bids (probability 0.3) or if there are competing bids but the lowest of these is greater than \$120,000 (probability $0.7(0.3 + 0.1)$). In this case the total probability that SciTools wins the bid is $0.3 + 0.28 = 0.58$.

TABLE 10.3 **Alternative Payoff Table for SciTools Bidding Example**

		Monetary value		
		SciTools Wins	**SciTools Loses**	**Probability That SciTools Wins**
SciTools'	**No bid**	NA	0	0.00
Bid	**115**	15	−5	0.86
(\$1000s)	**120**	20	−5	0.58
	125	25	−5	0.37

Risk Profiles From Table 10.3 we can obtain **risk profiles** for each of SciTools' decisions. A risk profile simply lists all possible monetary values and their corresponding probabilities. For example, if SciTools bids \$120,000, there are two monetary values possible, a profit of \$20,000 or a loss of \$5000, and their probabilities are 0.58 and 0.42, respectively. On the other hand, if SciTools decides not to bid, there is a sure monetary value of \$0—no profit, no loss.

A risk profile can also be illustrated graphically as a bar chart. There is a bar above each possible monetary value with height proportional to the probability of that value. For example, the risk profile for a \$120,000 bid decision is a bar chart with two bars, one above −\$5000 with height 0.42 and one above \$20,000 with height 0.58. The risk profile for the "no bid" decision is even simpler. It has a single bar above \$0 with height 1. We have not shown these bar charts for this example because they are so simple, but in more complex examples they can provide very useful information.

Expected Monetary Value (EMV) From the information we have discussed so far, it is not at all obvious which decision SciTools should make. The "no bid" decision is certainly safe, but it is certain to make zero profit. If SciTools decides to bid, the probability that it will lose \$5000 is smallest with the \$115,000 bid, but this bid has the smallest potential profit. Of course, if SciTools knew what the competitors were going to do, its decision would be easy. However, this uncertainty is the defining aspect of the problems in this chapter. The decision must be made *before* the uncertainty is resolved.

The most common way to make the choice is to calculate the **expected monetary value (EMV)** of each alternative and then choose the alternative with the largest EMV. The EMV is a weighted average of the possible monetary values, weighted by their probabilities. Formally, if v_i is the monetary value corresponding to outcome i and p_i is its probability, then EMV is defined as

$$\text{EMV} = \sum v_i p_i$$

In words, EMV is the mean of the probability distribution of possible monetary outcomes.

TABLE 10.4 EMVs for SciTools Bidding Example

Alternative	EMV Calculation	EMV
No bid	$0(1)$	\$0
Bid \$115,000	$15{,}000(0.86) + (-5000)(0.14)$	\$12,200
Bid \$120,000	$20{,}000(0.58) + (-5000)(0.42)$	\$9,500
Bid \$125,000	$25{,}000(0.37) + (-5000)(0.63)$	\$6,100

The EMVs for SciTools' problem are listed in Table 10.4. They indicate that if SciTools uses the EMV criterion for making its decision, it should bid \$115,000, as this yields the largest EMV.

It is very important to understand what an EMV implies and what it does not imply. If SciTools bids \$115,000, then its EMV is \$12,200. However, SciTools will certainly *not* earn a profit of \$12,200. It will earn \$15,000 or it will lose \$5000. So what does the EMV of \$12,200 really mean? It means that if SciTools could enter many "gambles" like this, where on each gamble it would win \$15,000 with probability 0.86 or lose \$5000 with probability 0.14, then *on average* it would win \$12,200 per gamble. In other words, the EMV can be interpreted as a long-term average.

It might seem peculiar, then, to base a one-time decision on EMV, which represents a long-term average. There are two ways to explain this apparent inconsistency. First, most companies make frequent decisions under uncertainty. Although each decision might have its own unique characteristics, it seems reasonable that if the company plans to make many such decisions, it should be willing to "play the averages," as it does when it uses EMV as the decision criterion. Second, even if this is the only such decision the company is *ever* going to make, decision theorists have proven that under certain conditions, maximizing EMV is a rational basis for making this decision. These "certain conditions" relate to the decision maker's attitude toward risk. As we will discuss later in this chapter, if the decision maker is risk averse and the possible monetary payoffs or losses are large relative to her wealth, then EMV is *not* the appropriate decision criterion to use. However, the EMV criterion has proved useful in the vast majority of decision-making applications, so we will use it throughout most of this chapter.

Decision Trees By now, we have gone through most of the steps of solving SciTools' problem. We have listed the decision alternatives, the uncertain outcomes and their probabilities, and the profits and losses from all combinations of decisions and outcomes. We have then calculated the EMV for each alternative and have chosen the alternative with the largest EMV. All of this can be done efficiently using a graphical tool called a **decision tree**. The decision tree that corresponds to SciTools' problem appears in Figure 10.1 (page 500). (This figure is actually part of an Excel spreadsheet and was created with the PrecisionTree add-in. We will explain how it was created shortly.)

Decision Tree Conventions To understand Figure 10.1, we need to know the following conventions that have been established for decision trees.

1. Decision trees are composed of **nodes** (circles, squares, and triangles) and **branches** (lines).
2. The nodes represent points in time. A **decision node** (a square) is a time when the decision maker makes a decision. A **probability node** (a circle) is a time when the result of an uncertain event becomes known. An **end node** (a triangle) indicates

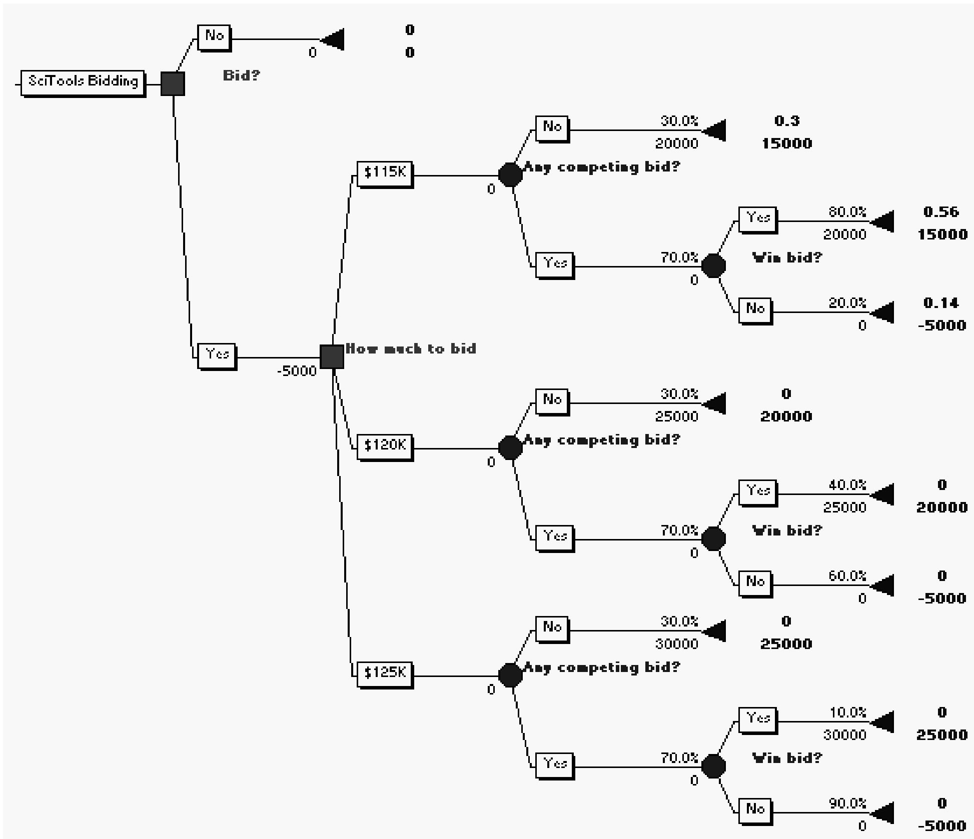


FIGURE 10.1 Decision Tree for SciTools Bidding Example

that the problem is completed—all decisions have been made, all uncertainty has been resolved, and all payoffs/costs have been incurred.

3. Time proceeds *from left to right*. This means that any branches leading into a node (from the left) have already occurred. Any branches leading out of a node (to the right) have not yet occurred.
4. Branches leading out of a decision node represent the possible decisions; the decision maker can choose the preferred branch. Branches leading out of probability nodes represent the possible outcomes of uncertain events; the decision maker has no control over which of these will occur.
5. Probabilities are listed on probability branches. These probabilities are *conditional* on the events that have already been observed (those to the left). Furthermore, the probabilities on branches leading out of any particular probability node must sum to 1.
6. Individual monetary values are shown on the branches where they occur, and cumulative monetary values are shown to the right of the end nodes. (Actually, PrecisionTree shows two values to the right of each end node. The top one is the probability of getting to that end node, and the bottom one is the associated monetary value.)

The decision tree in Figure 10.1 illustrates these conventions for a **single-stage** decision problem, the simplest type of decision problem. In a single-stage problem all decisions are made *first*, and then all uncertainty is resolved. Later in this chapter we will see **multistage** decision problems, where decisions and outcomes alternate. That is, a decision maker makes a decision, then some uncertainty is resolved, then the decision maker makes a second decision, then some further uncertainty is resolved, and so on. Because these multistage decisions problems are inherently more complex, we will focus initially on single-stage problems.

Once a decision tree has been drawn and labeled with probabilities and monetary values, it can be solved easily. The solution for the decision tree in Figure 10.1 is shown in Figure 10.2. Among other things, it shows that the decision to bid \$115,000 is optimal (follow the decision branches with "True" above them), with a corresponding EMV of \$12,200 (the value under "Bid?" at the left of the tree). This is consistent with what we saw earlier for this example.

Folding Back Procedure The solution procedure used to develop Figure 10.2 is called **folding back** on the tree. Starting at the right of the tree and working back to the left, the procedure consists of two types of calculations.

1. At each probability node, we calculate the EMV (sum of monetary values times probabilities) and write it below the name of the node. For example, consider the node (top right) after SciTools' decision to bid \$115,000 and after it learns that

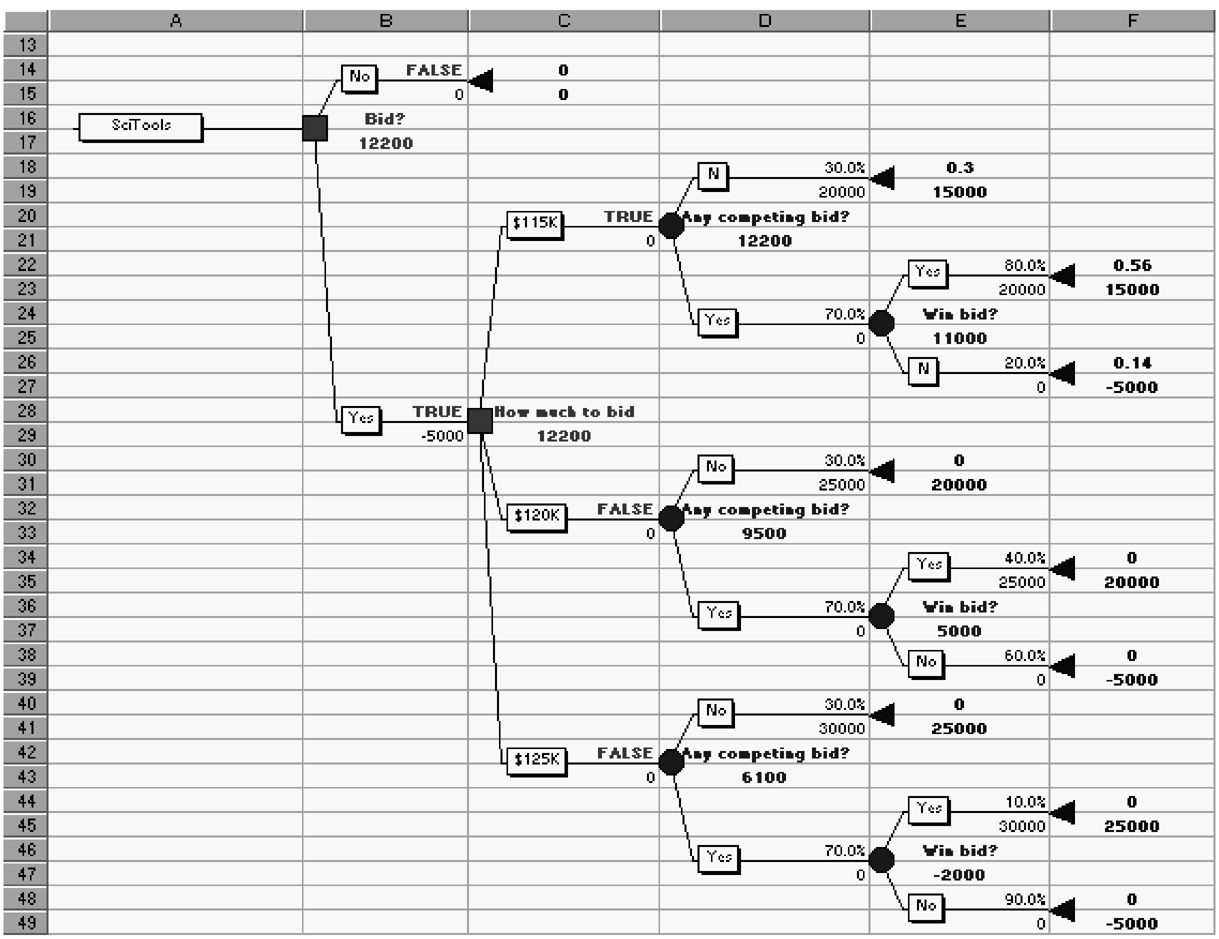


FIGURE 10.2 Result of Folding Back to Obtain Optimal Decision

there will be a competing bid. From that point, SciTools will either win \$15,000 with probability 0.8 or lose \$5000 with probability 0.2. The corresponding EMV is

$$0.8(15{,}000) + 0.2(-5000) = 11{,}000$$

and this value is entered below the node name "Win bid?".

Now, back up a step and consider the preceding probability node (the one to the left of the "Win bid?" node). At this point, SciTools has bid \$115,000 and is about to discover whether there will be a competing bid. If there is none, with probability 0.3, then SciTools will win \$15,000. But if there is a competing bid, with probability 0.7, the EMV from that point on is the \$11,000 we just calculated. Essentially, this \$11,000 summarizes the consequences of being at the "Win bid?" node, and SciTools acts the same as if it were going to receive \$11,000 *for certain*. Therefore, the EMV for the "Any competing bid?" node is

$$0.3(15{,}000) + 0.7(11{,}000) = 12{,}200$$

This EMV is written below the node name.

2. Decision nodes are much easier. At each decision node we find the maximum of the EMVs and write it below the node name. PrecisionTree indicates the winner by placing "True" on the decision branch with the maximum EMV and "False" on all other branches emanating from this node. For example, consider the node where SciTools is deciding how much to bid (after already having decided to place a bid). The EMVs under the three succeeding *probability* nodes are \$12,200, \$9500, and \$6100. Since the maximum of these is \$12,200, the EMV for the "How much to bid" node is \$12,200 and is written below the node name.

After the folding-back process is completed—that is, after we have calculated EMVs for all nodes—we can trace the "True" labels from left to right to see the optimal strategy. In this case SciTools should place a bid, and it should be for \$115,000. The EMV written below the leftmost decision node, \$12,200, indicates SciTools' EMV for this strategy. If SciTools is truly willing to use the EMV criterion, that is, if it is willing to play the averages, then the company should be indifferent between receiving \$12,200 *for certain* and bidding \$115,000—with the associated risk of winning \$15,000 or losing \$5000.

The PrecisionTree Add-In Decision trees present a challenge for Excel. We must somehow take advantage of Excel's calculating capabilities (to calculate EMVs, for example) and its graphical capabilities (to depict the decision tree). Fortunately, there is now a powerful add-in, PrecisionTree developed by Palisade Corporation, that makes the process relatively straightforward.[2] This add-in not only enables us to build and label a decision tree, but it performs the folding-back procedure automatically and then allows us to perform sensitivity analysis on key input parameters.

The first thing you must do to use PrecisionTree is to "add it in." You do this in two steps. First, you must install the Palisade Decision Tools suite (or at least the PrecisionTree program) with the Setup program on the CD-ROM accompanying this book. Of course, you need to do this only once. Then to run PrecisionTree, there are three options:

[2]The educational version of PrecisionTree included with this book is slightly scaled down from Palisade's commercial version of PrecisionTree. The difference you are most likely to notice is that the educational version permits only 50 nodes—of all types combined—in a decision tree.

- If Excel is not currently running, you can launch Excel *and* PrecisionTree by clicking on the Windows Start button and selecting the PrecisionTree item from the Palisade Decision Tools group of the Programs group.
- If Excel is currently running, the procedure in the previous bullet will launch PrecisionTree on top of Excel.
- If Excel is already running and the Decision Tools toolbar in Figure 10.3 is showing, you can start PrecisionTree by clicking on its icon (the third from the left).

FIGURE 10.3
Palisade Decision Tools Toolbar

You will know that PrecisionTree is ready for use when you see its toolbar (shown in Figure 10.4) and a PrecisionTree menu to the left of the Help menu. By the way, if you want to unload PrecisionTree *without* closing Excel, use the PrecisionTree/Help/About menu item and click on Unload. It's a bit unconventional, but it works.

Using PrecisionTree PrecisionTree is quite easy to use—at least its most basic items are—but it can be confusing at first. We will lead you through the steps for the SciTools example. (The file SCITOOLS.XLS shows the results of this procedure, but you should work through the steps on your own, starting with a blank spreadsheet.)

1. **Inputs.** Enter the inputs shown in columns A and B of Figure 10.5.
2. **New tree.** Click on the new tree button (the far left button) on the PrecisionTree toolbar, and then click on any cell (say, cell A14) below the input section to start a new tree. Click on the name box of this new tree (it probably says "tree #1") to open a dialog box. Type in a descriptive name for the tree, such as SciTools Bidding, and click on OK. You should now see the beginnings of a tree, as shown in Figure 10.6 (page 504).
3. **Decision nodes and branches.** From here on, keep the finished tree in Figure 10.2 in mind. This is the finished product we eventually want. To obtain decision nodes

FIGURE 10.4
PrecisionTree Toolbar

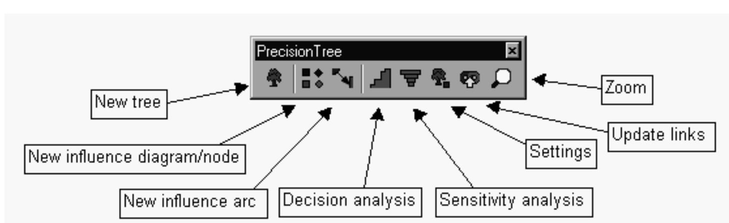
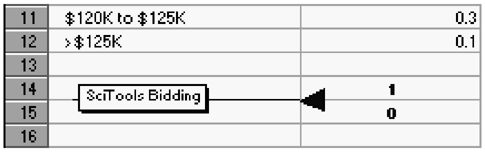


FIGURE 10.5
Inputs for SciTools Bidding Example

	A	B	C
1	**SciTools Bidding Example**		
2			
3	**Inputs**		
4	Cost to prepare a bid	$5,000	
5	Cost to supply instruments	$95,000	
6			
7	Probability of no competing bid	0.3	
8	Comp bid distribution (if they		
9	<$115K	0.2	
10	$115K to $120K	0.4	
11	$120K to $125K	0.3	
12	>$125K	0.1	

Range names
BidCost: B4
PrNoBid: B7
ProdCost: B5

FIGURE 10.6
Beginnings of a New Tree



and branches, click on the (only) triangle end node to open the dialog box in Figure 10.7. Click on the green square to indicate that this is a decision node, and fill in the dialog box as shown. We're calling this decision "Bid?" and specifying that there are two possible decisions. The tree expands as shown in Figure 10.8. The boxes that say "branch" show the default labels for these branches. Click on either of them to open another dialog box where you can provide a more descriptive name for the branch. Do this to label the two branches "No" and "Yes." Also, you can enter the immediate payoff/cost for either branch right below it. Since there is a $5000 cost of bidding, enter the formula

=-BidCost

right below the "Yes" branch in cell B19. (It is negative to reflect a *cost*.) The tree should now appear as in Figure 10.9.

4. **More decision branches.** The top branch is completed; if SciTools does not bid, there is nothing left to do. So click on the bottom end node, following SciTools' decision to bid, and proceed as in the previous step to add and label the decision

FIGURE 10.7
Dialog Box for Adding a New Decision Node and Branches

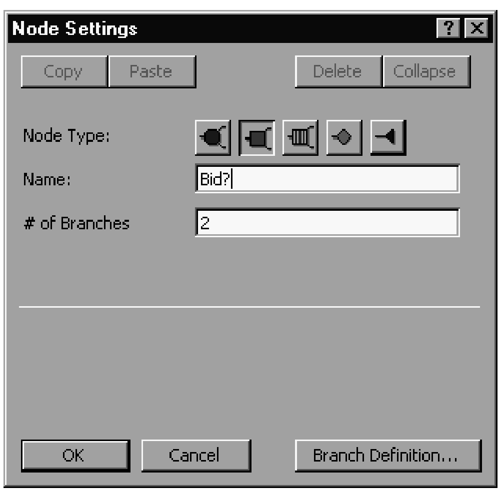


FIGURE 10.8
Tree with Initial Decision Node and Branches

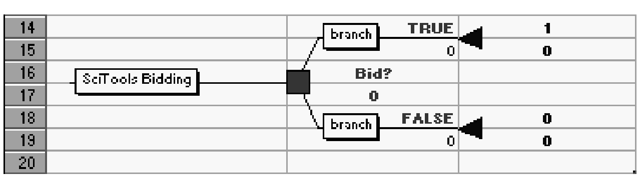
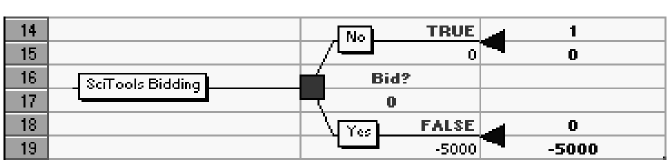


FIGURE 10.9
Decision Tree with Decision Branches Labeled



node and three decision branches for the amount to bid. (Refer to Figure 10.2.) The tree to this point should appear as in Figure 10.10. Note that there are no monetary values below these decision branches because no *immediate* payoffs or costs are associated with the bid amount decision.

5. **Probability nodes and branches.** We now need a probability node and branches from the rightmost end nodes to capture whether the competition bids. Click on the top one of these end nodes to bring up the same dialog box as in Figure 10.7. Now, however, click on the red circle box to indicate that this is a probability node. Label it “Any competing bid?”, specify two branches, and click on OK. Then label the two branches “No” and “Yes.” Next, repeat this procedure to form another probability node (with two branches) following the “Yes” branch, call it “Win bid?”, and label its branches as shown in Figure 10.11.
6. **Copying probability nodes and branches.** You could now repeat the same procedure from the previous step to build probability nodes and branches following the other bid amount decisions, but because they’re structurally equivalent, you can save a lot of work by using PrecisionTree’s copy and paste feature. Click on the leftmost probability node to open a dialog box and click on Copy. Then click on either end node to bring up the same dialog box and click on Paste. Do this again with the other end node. Decision trees can get very “bushy,” but this copy and paste feature can make them much less tedious to construct.

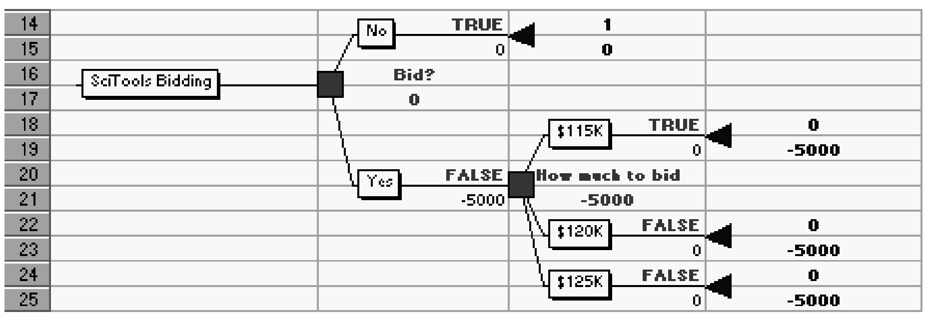


FIGURE 10.10 Tree with All Decision Nodes and Branches

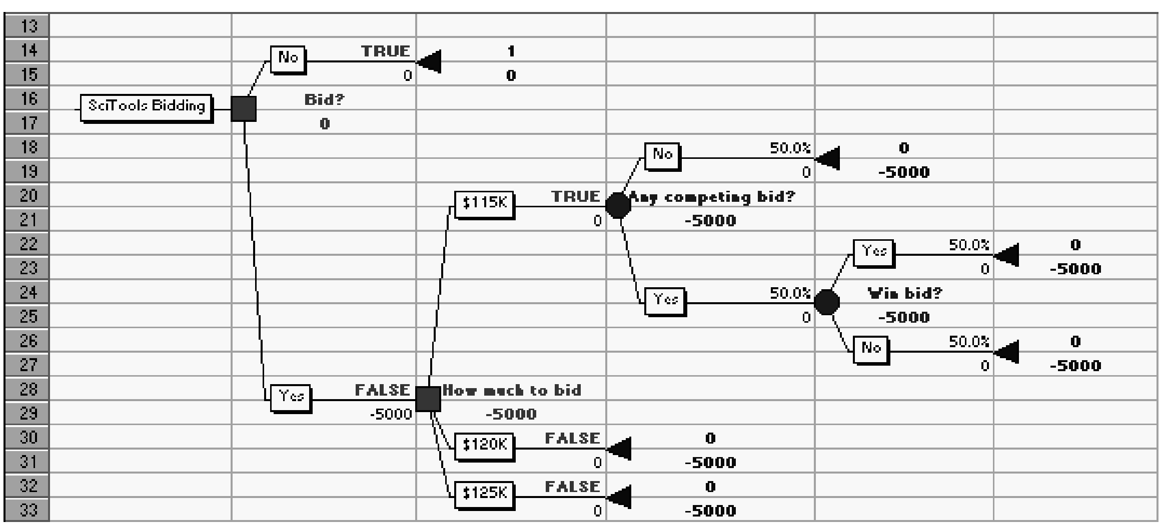


FIGURE 10.11 Decision Tree with One Set of Probability Nodes and Branches

7. **Labeling probability branches.** You should now have the decision tree shown in Figure 10.12. It is structurally the same as the completed tree in Figure 10.2, but the probabilities and monetary values on the probability branches are not correct. Note that each probability branch has a value above and below the branch. The value above is the probability (the default values make the branches equally likely), and the value below is the monetary value (the default values are 0). We can enter any values or formulas in these cells, exactly as we do in typical Excel worksheets. As usual, it is a good practice to refer to input cells in these formulas whenever possible. We'll get you started with the probability branches following the decision to bid $115,000. First, enter the probability of no competing bid in cell D18 with the formula

   =PrNoBid

   and enter its complement in cell D24 with the formula

   =1-D18

   Next, enter the probability that SciTools wins the bid in cell E22 with the formula

   =SUM(B10:B12)

   and enter its complement in cell E26 with the formula

   =1-E22

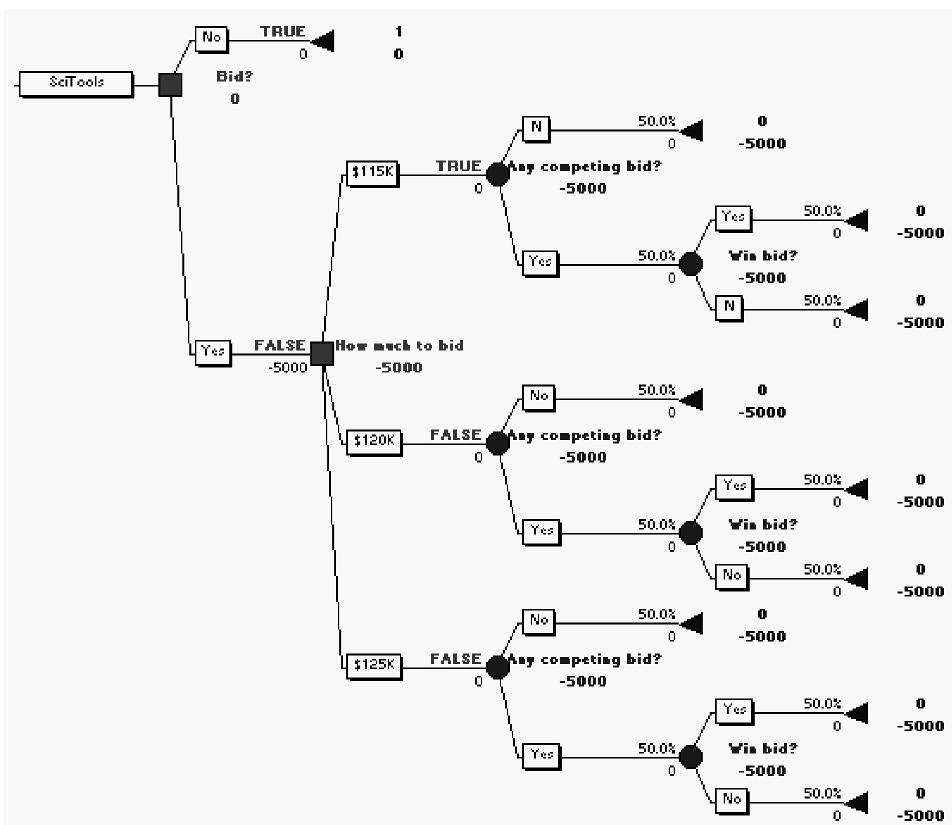


FIGURE 10.12 Structure of Completed Tree

(Remember that SciTools wins the bid only if the competitor bids higher, and in this part of the tree, SciTools is bidding $115,000.) For the monetary values, enter the formula

=115000-ProdCost

in the two cells, D19 and E23, where SciTools wins the contract. Note that we already subtracted the cost of the bid (cell B29), so we shouldn't do so again. This would be double-counting, and it should always be avoided in decision problems.

8. **Enter the other formulas on probability branches.** Using the previous step and Figure 10.2 as a guide, enter formulas for the probabilities and monetary values on the other probability branches, that is, those following the decision to bid $120,000 or $125,000.

We're finished! The completed tree in Figure 10.2 shows the best strategy and its associated EMV, as we discussed earlier. Note that we never have to perform the folding-back procedure manually. PrecisionTree does it for us. In fact, the tree is completed as soon as we finish entering the relevant inputs. In addition, if we change any of the inputs, the tree reacts automatically. For example, try changing the bid cost in cell B4 from $5000 to some large value such as $20,000. You'll see that the tree calculations update automatically, and the best decision is then *not* to bid, with an associated EMV of $0.

Risk Profile of Optimal Strategy Once the decision tree is completed, PrecisionTree has several tools we can use to gain more information about the decision analysis. First, we can see a risk profile and other information about the *optimal* decision. To do so, click on the fourth button from the left on the PrecisionTree toolbar (it looks like a staircase) and fill in the resulting dialog box as shown in Figure 10.13. (You can experiment with other options.) The Policy Suggestion option allows us to see only that part of the tree that corresponds to the best decision, as shown in Figure 10.14 (page 508).

The Risk Profile option allows us to see a graphical risk profile of the optimal decision. (If we checked the Statistics Report box, we would also see this information numerically.) As the risk profile in Figure 10.15 (page 508) shows, there are only two possible monetary outcomes if SciTools bids $115,000. It either wins $15,000 or loses $5000, and the former is much more likely. (The associated probabilities are 0.86 and 0.14.) This graphical information is even more useful when there are a larger number of possible monetary outcomes. We can see what they are and how likely they are.

FIGURE 10.13
Dialog Box for Information About Optimal Decision

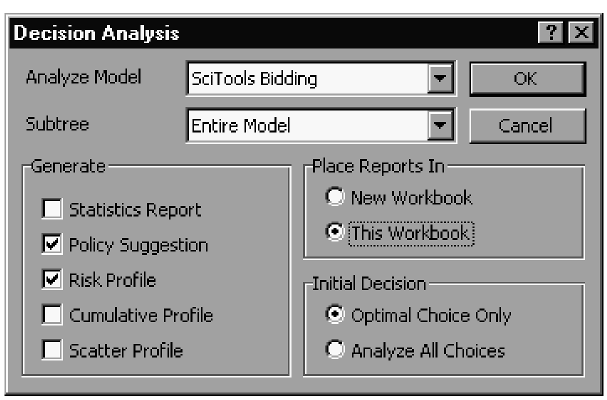


FIGURE 10.14
Subtree for Optimal Decision

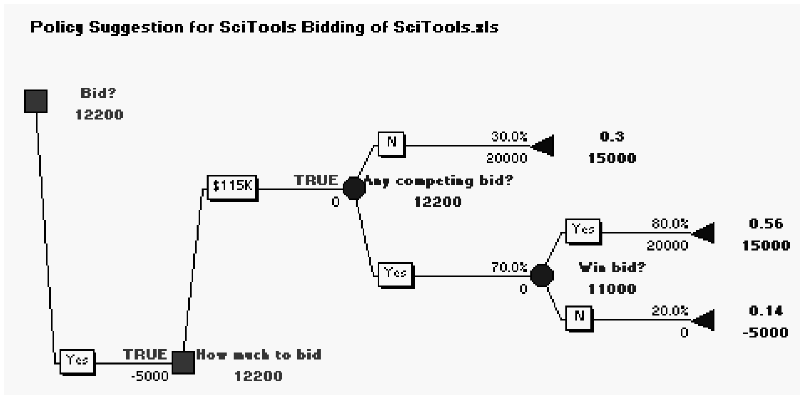
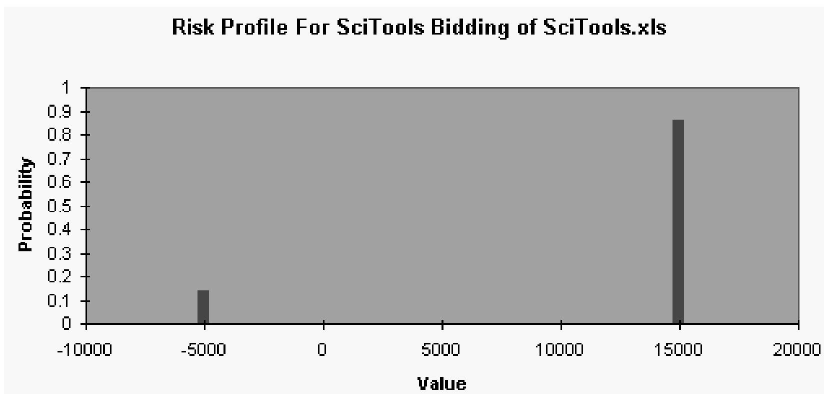


FIGURE 10.15
Risk Profile of Optimal Decision



Sensitivity Analysis We have already stressed the importance of a follow-up sensitivity analysis for any decision problem, and PrecisionTree makes this relatively easy to perform. First, we can enter any values into the input cells and watch how the tree changes. But we can get more systematic information by clicking on PrecisionTree's sensitivity button, the fifth from the left on the toolbar (it looks like a tornado). This brings up the dialog box in Figure 10.16. It requires an EMV cell (and an optional descriptive name) to analyze at the top and one or more input cells in the middle. The specifications for these input cells are actually entered at the bottom of the dialog box.

The cell to analyze (at the top) is usually the EMV cell at the far left of the decision tree—this is the cell shown in the figure—but it can be any EMV cell. For example, if we *assume* SciTools will prepare a bid and we want to see how sensitive the EMV from that point on is to inputs, we could select cell C29 (refer to Figure 10.2) to analyze. Next, for any input cell such as the production cost cell (B5), we enter a minimum value, a maximum value, a base value (probably the original value in the model), and a step size. For example, to specify these for the production cost, we clicked on the Suggest Values button. This default setting varies the production cost by as much as 10% from the original value in either direction in a series of 10 steps. We can also enter our own desired values. We did so for the probability of no competing bids, varying its value from 0 to 0.6 in a sequence of 12 steps.

When we click on Run Analysis, PrecisionTree varies each of the specified inputs (one at a time if we select the One Way option) and presents the results in several ways in a *new* Excel file with Sensitivity, Tornado, and Spider Graph sheets. The Sensitivity sheet includes several charts, a typical one of which appears in Figure 10.17. This shows how the EMV varies with the production cost for *both* of the original decisions

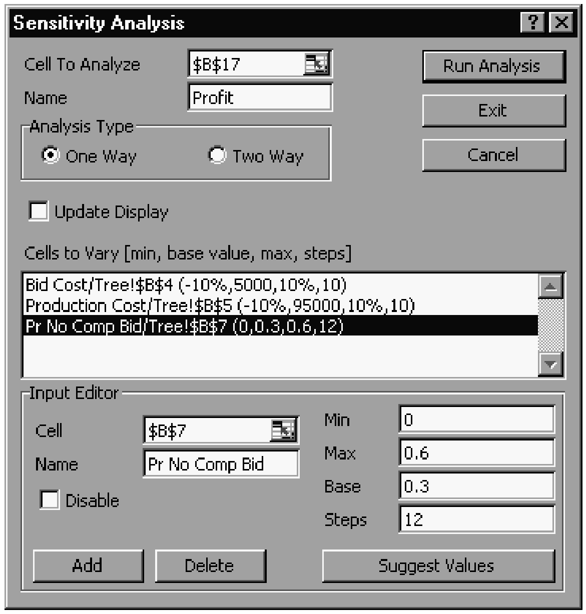


FIGURE 10.16
Sensitivity Analysis Dialog Box

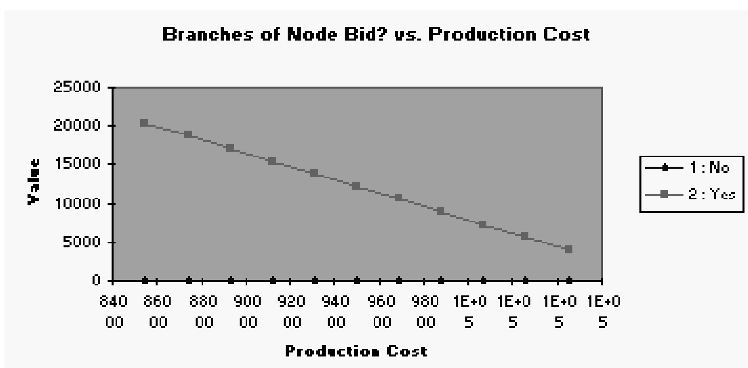


FIGURE 10.17
EMV versus Production Cost for Each of Two Decisions

(bid or don't bid). This type of graph is useful for seeing whether the optimal decision *changes* over the range of the input variable. It does so only if the two lines cross. In this particular graph it is clear that the "Bid" decision dominates the "No bid" decision over the production cost range we selected.

The Tornado sheet shows how sensitive the EMV of the *optimal* decision is to each of the selected inputs over the ranges selected. (See Figure 10.18 (page 510).) The length of each bar shows the percentage change in the EMV in either direction, so the longer the bar, the more sensitive this EMV is to the particular input. The bars are always arranged from longest on top to shortest on the bottom—hence the name *tornado* chart. Here we see that production cost has the largest effect on EMV, and bid cost has the smallest effect.

Finally, the Spider Chart sheet contains the chart in Figure 10.19. It shows how much the optimal EMV varies in magnitude for various percentage changes in the input variables. The steeper the slope of the line, the more the EMV is affected by a particular input. We again see that the production cost has a relatively large effect, whereas the other two inputs have relatively small effects.

Each time we click on the sensitivity button, we can run a different sensitivity analysis. An interesting option is to run a two-way analysis (by clicking on the Two

FIGURE 10.18
Tornado Chart for SciTools Example

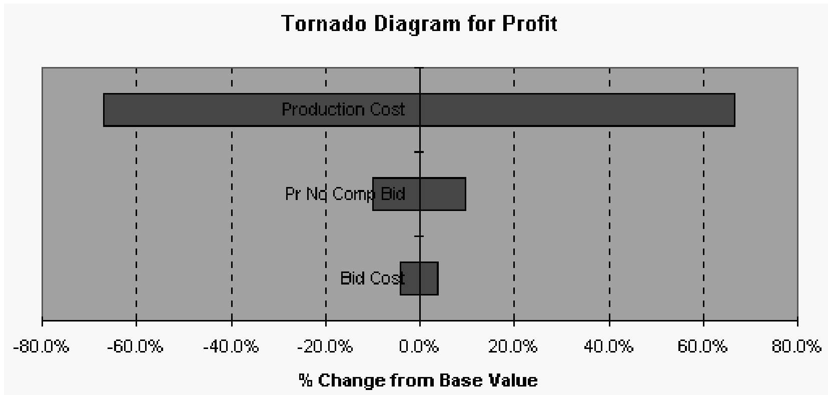
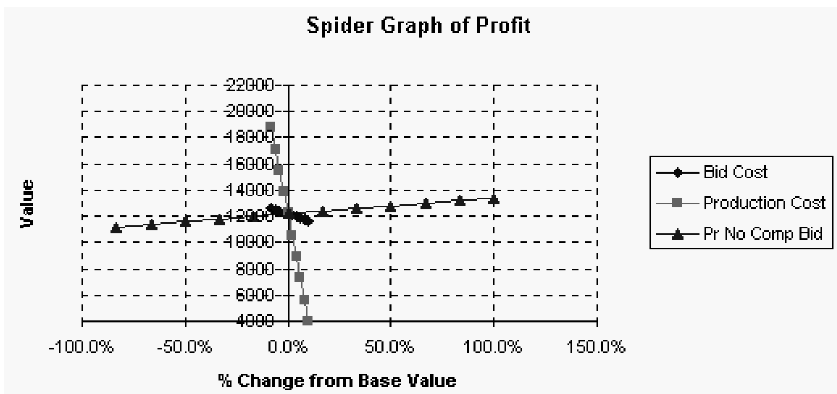
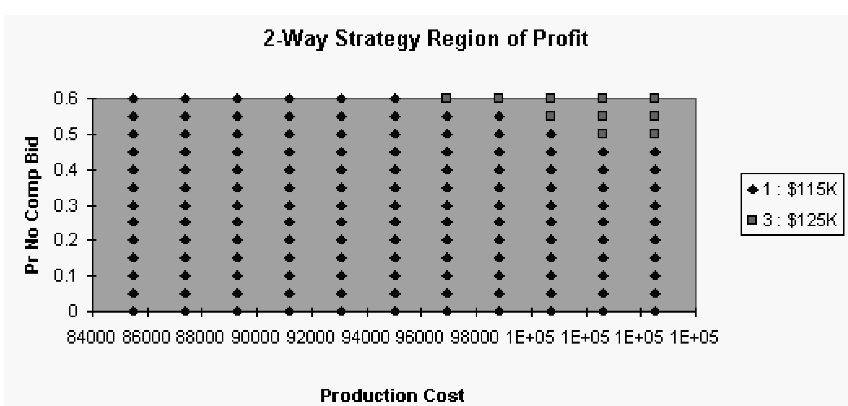


FIGURE 10.19
Spider Chart for SciTools Example



Way button in Figure 10.16). Then we see how the selected EMV varies as each *pair* of inputs vary simultaneously. We analyzed the EMV in cell C29 with this option, using the same inputs as before. A typical result is shown in Figure 10.20. For each of the possible values of production cost and the probability of no competitor bid, this chart indicates which bid amount is optimal. (By choosing cell C29, we are assuming SciTools will bid; the question is only how much.) As we see, the optimal bid amount remains $115,000 unless the production cost *and* the probability of no competing bid are both large. Then it becomes optimal to bid $125,000. This makes sense intuitively. As the chance of no competing bid increases and a larger production cost must be recovered, it seems reasonable that SciTools should increase its bid.

FIGURE 10.20
Two-Way Sensitivity Analysis



We reiterate that a sensitivity analysis is always an important aspect in real decision analyses. If we had to construct decision trees by hand—with paper and pencil—a sensitivity analysis would be virtually out of the question. We would have to recompute everything each time through. Therefore, one of the most valuable features of the PrecisionTree add-in is that it enables us to perform sensitivity analyses in a matter of seconds. ■

PROBLEMS

Skill-Building Problems

1. The SweetTooth Candy Company knows it will need 10 tons of sugar 6 months from now to implement its production plans. Jean Dobson, SweetTooth's purchasing manager, has essentially two options for acquiring the needed sugar. She can either buy the sugar at the going market price when she needs it, 6 months from now, or she can buy a futures contract now. The contract guarantees delivery of the sugar in 6 months but the cost of purchasing it will be based on today's market price. Assume that possible sugar futures contracts available for purchase are for 5 tons or 10 tons only. No futures contracts can be purchased or sold in the intervening months. Thus, SweetTooth's possible decisions are: (1) purchase a futures contract for 10 tons of sugar now, (2) purchase a futures contract for 5 tons of sugar now and purchase 5 tons of sugar in 6 months, or (3) purchase all 10 tons of needed sugar in 6 months. The price of sugar bought now for delivery in 6 months is $0.0851 per pound. The transaction costs for 5-ton and 10-ton futures contracts are $65 and $110, respectively. Finally, Ms. Dobson has assessed the probability distribution for the possible prices of sugar 6 months from now (in dollars per pound). Table 10.5 contains these possible prices and their corresponding probabilities.
   - **a.** Given that SweetTooth wants to acquire the needed sugar in the least-cost way, formulate a payoff table that specifies the cost (in dollars) associated with each possible decision and possible sugar price in the future.
   - **b.** Use the PrecisionTree add-in to identify the strategy that minimizes SweetTooth's expected cost of meeting its sugar demand. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected cost value more sensitive?
   - **c.** Generate a risk profile for SweetTooth's optimal decision.

TABLE 10.5 Distribution of Possible Sugar Prices

Possible Sugar Prices in 6 Months ($/pound)	Probability
0.078	0.05
0.083	0.25
0.087	0.35
0.091	0.20
0.096	0.15

2. Carlisle Tire and Rubber, Inc. is considering expanding production to meet potential increases in the demand for one of its tire products. Carlisle's alternatives are to construct a new plant, expand the existing plant, or do nothing in the short run. The market for this particular tire product may expand, remain stable, or contract. Carlisle's marketing department estimates the probabilities of these market outcomes as 0.25, 0.35, and 0.40, respectively. Table 10.6 contains Carlisle's estimated payoff (in dollars) table.
   - **a.** Use the PrecisionTree add-in to identify the strategy that maximizes this tire manufacturer's expected profit. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected profit value most sensitive?

TABLE 10.6 Payoff Table for Carlisle's Decision Problem

Decision/Market Outcome	Market Expands	Market Stable	Market Contracts
Construct a new plant	400,000	−100,000	−200,000
Expand existing plant	250,000	−50,000	−75,000
Do nothing	50,000	0	−30,000

b. Generate a risk profile for Carlisle's optimal decision.

3. A local energy provider offers a landowner $180,000 for the exploration rights to natural gas on a certain site and the option for future development. This option, if exercised, is worth an additional $1,800,000 to the landowner, but this will occur only if natural gas is discovered during the exploration phase. The landowner, believing that the energy company's interest in the site is a good indication that gas is present, is tempted to develop the field herself. To do so, she must contract with local experts in natural gas exploration and development. The initial cost for such a contract is $300,000, which is lost forever if no gas is found on the site. If gas is discovered, however, the landowner expects to earn a net profit of $6,000,000. Finally, the landowner estimates the probability of finding gas on this site to be 60%.

a. Formulate a payoff table that specifies the landowner's payoff (in dollars) associated with each possible decision and each outcome with respect to finding natural gas on the site.

b. Use the PrecisionTree add-in to identify the strategy that maximizes the landowner's expected net earnings from this opportunity. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected profit value most sensitive?

c. Generate a risk profile for landowner's optimal decision.

4. Techware Incorporated is considering the introduction of two new software products to the market. In particular, the company has four options regarding these two proposed products: introduce neither product, introduce product 1 only, introduce product 2 only, or introduce both products. Research and development costs for products 1 and 2 are $180,000 and $150,000, respectively. Note that the first option entails no costs because research and development efforts have not yet begun. The success of these software products depends on the trend of the national economy in the coming year and on the consumers' reaction to these products. The company's revenues earned by introducing product 1 only, product 2 only, or both products in various states of the national economy are given in Table 10.7. The probabilities of observing a strong, fair, and weak trend in the national economy in the coming year are 0.30, 0.50, and 0.20, respectively.

a. Formulate a payoff table that specifies Techware's net revenue (in dollars) for each possible decision and each outcome with respect to the trend in the national economy.

b. Use the PrecisionTree add-in to identify the strategy that maximizes Techware's expected net revenue from the given marketing opportunities. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected net revenue value most sensitive?

c. Generate a risk profile for Techware's optimal decision.

5. Consider an investor with $10,000 available to invest. He has the following options regarding the allocation of his available funds: (1) he can invest in a risk-free savings account with a guaranteed 3% annual rate of return; (2) he can invest in a fairly safe stock, where the possible annual rates of return are 6%, 8%, or 10%; or (3) he can invest in a more risky stock where the possible annual rates of return are 1%, 9%, or 17%. Note that the investor can place all of his available funds in any one of these options, or he can split his $10,000 into two $5000 investments in any two of these options. The joint probability distribution of the possible return rates for the two stocks is given in Table 10.8.

a. Formulate a payoff table that specifies this investor's return (in dollars) in one year for each possible decision and each outcome with respect to the two stock returns.

b. Use the PrecisionTree add-in to identify the strategy that maximizes the investor's expected earnings in one year from the given investment opportunities. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected earnings value most sensitive?

c. Generate a risk profile for this investor's optimal decision.

6. A buyer for a large department store chain must place orders with an athletic shoe manufacturer 6 months prior to the time the shoes will be sold in the department stores. In particular, the buyer must

TABLE 10.7 Revenue Table for Techware's Decision Problem

Decision/Trend in National Economy	Strong	Fair	Weak
Introduce neither product	$0	$0	$0
Introduce product 1 only	$500,000	$260,000	$120,000
Introduce product 2 only	$420,000	$230,000	$110,000
Introduce both products	$820,000	$390,000	$200,000

TABLE 10.8 **Joint Probability Distribution of Safe and Risky Stock Return Rates**

Safe Stock Return Rates (S)/ Risky Stock Return Rates (R)	$R = 1\%$	$R = 9\%$	$R = 17\%$
$S = 6\%$	0.10	0.05	0.10
$S = 8\%$	0.25	0.05	0.20
$S = 10\%$	0.10	0.05	0.10

decide on November 1 how many pairs of the manufacturer's newest model of tennis shoes to order for sale during the upcoming summer season. Assume that each pair of this new brand of tennis shoes costs the department store chain $45 per pair. Furthermore, assume that each pair of these shoes can then be sold to the chain's customers for $70 per pair. Any pairs of these shoes remaining unsold at the end of the summer season will be sold in a closeout sale next fall for $35 each. The probability distribution of consumer demand for these tennis shoes (in hundreds of pairs) during the upcoming summer season has been assessed by market research specialists and is provided in Table 10.9. Finally, assume that the department store chain must purchase these tennis shoes from the manufacturer in lots of 100 pairs.

TABLE 10.9 **Distribution of Consumer Demand for Tennis Shoes**

Consumer Demand	Probability
1	0.025
2	0.050
3	0.075
4	0.100
5	0.150
6	0.200
7	0.175
8	0.100
9	0.075
10	0.050

a. Formulate a payoff table that specifies the contribution to profit (in dollars) from the sale of the tennis shoes by this department store chain for each possible purchase decision (in hundreds of pairs) and each outcome with respect to consumer demand.

b. Use the PrecisionTree add-in to identify the strategy that maximizes the department store chain's expected profit earned by purchasing and subsequently selling pairs of the new tennis shoes. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected earnings value most sensitive?

c. Generate a risk profile for the buyer's optimal decision.

Skill-Extending Problems

7. In designing a new space vehicle, NASA needs to decide whether to provide 0, 1, or 2 backup systems for a critical component of the vehicle. The first backup system, if included, comes into use only if the original system fails. The second backup system, if included, comes into use only if the original system and the first backup system both fail. NASA engineers claim that each system, independently of the others, has a 1% chance of failing if called into use. Each backup system costs $70,000 to produce and install within the vehicle. Once the vehicle is in flight, the mission will be scrubbed only if the original system and all backups fail. The cost of a scrubbed mission, in addition to production costs, is assessed to be $8,000,000.

a. Use the PrecisionTree add-in to identify the strategy that minimizes NASA's expected total cost. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected earnings value most sensitive?

b. Generate a risk profile for NASA's optimal decision.

8. Mr. Maloy has just bought a new $30,000 sport utility vehicle. As a reasonably safe driver, he believes that there is only about a 5% chance of being in an accident in the forthcoming year. If he is involved in an accident, the damage to his new vehicle depends on the severity of the accident. The probability distribution for the range of possible accidents and the corresponding damage amounts (in dollars) are given in Table 10.10 (page 514). Mr. Maloy is trying to decide whether he is willing to pay $170 each year for collision insurance with a $300 deductible. Note that with this type of insurance, he pays the *first* $300 in damages if he causes an accident and the insurance company pays the remainder.

TABLE 10.10 **Distribution of Accident Types and Corresponding Damage Amounts**

Type of Accident	Conditional Probability	Damage to Vehicle
Minor	0.60	$200
Moderate	0.20	$1,000
Serious	0.10	$4,000
Catastrophic	0.10	$30,000

TABLE 10.11 **Distribution of Defective Components in a Lot**

Proportion of Defective Components	Probability
0.05	0.50
0.10	0.25
0.25	0.15
0.50	0.10

a. Formulate a payoff table that specifies the cost (in dollars) associated with each possible decision and type of accident.

b. Use the PrecisionTree add-in to identify the strategy that minimizes Mr. Maloy's annual expected total cost. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected earnings value most sensitive?

c. Generate a risk profile for Mr. Maloy's optimal decision.

9. The purchasing agent for a microcomputer manufacturer is currently negotiating a purchase agreement for a particular electronic component with a given supplier. This component is produced in lots of 1000, and the cost of purchasing a lot is $30,000. Unfortunately, past experience indicates that this supplier has occasionally shipped defective components to its customers. Specifically, the proportion of defective components supplied by this supplier is described by the probability distribution given in Table 10.11. While the microcomputer manufacturer can repair a defective component at a cost of $20 each, the purchasing agent is intrigued to learn that this supplier will now assume the cost of replacing defective components in excess of the first 100 faulty items found in a given lot. This guarantee may be purchased by the microcomputer manufacturer prior to the receipt of a given lot at a cost of $1000 per lot. The purchasing agent is interested in determining whether it is worthwhile for her company to purchase the supplier's guarantee policy.

a. Formulate a payoff table that specifies the microcomputer manufacturer's total cost (in dollars) of purchasing and repairing (if necessary) a complete lot of components for each possible decision and each outcome with respect to the proportion of defective items.

b. Use the PrecisionTree add-in to identify the strategy that minimizes the expected total cost of achieving a complete lot of satisfactory microcomputer components. Also, perform sensitivity analysis on the optimal decision and summarize your findings. In response to which model inputs is the expected earnings value most sensitive?

c. Generate a risk profile for the purchasing agent's optimal decision.

10.3 MORE SINGLE-STAGE EXAMPLES

All applications of decision making under uncertainty follow the procedures discussed so far. We first identify the possible decision alternatives, assess relevant probabilities, and calculate monetary values. Then we use a decision tree (or influence diagram) to identify the alternative with the largest EMV and follow this up with a thorough sensitivity analysis. We can also examine the risk profiles for the various alternatives. This is particularly useful if criteria other than EMV maximization are considered, as we will discuss in Section 7.8. In this section we will illustrate the process with several single-stage examples, where the decision maker makes one decision and then learns which of several uncertain outcomes occurs. In the next section we will examine multistage examples, where two or more sequential decisions must be made.

The following example illustrates a decision problem most of us face on an annual basis, although most of us probably do not go to the trouble of analyzing it formally.

EXAMPLE 10.2

SELECTING HEALTH CARE PLANS AT STATE UNIVERSITY

Each year employees at State University are asked to decide on one of three health care plans. The terms of these are as follows:[3]

Plan 1: The monthly cost is \$24. There is a \$500 deductible. The participant pays all expenses until payments for the year equal \$500. After that, 90% of remaining expenses are paid by the insurer.

Plan 2: This is the same as plan 1, except that the monthly cost is \$1 and the deductible amount is \$1000.

Plan 3: The monthly cost is \$20. There is no deductible. The employee pays 30% of all medical expenses. The rest is paid by the insurer.

Which of these three plans should an employee choose?

Solution

Clearly, the solution will vary from one employee to another, depending on the assessed probability distribution of medical expenses. To illustrate, however, we will consider an employee who assesses the distribution of yearly medical expenses shown in Table 10.12. These expenses include hospital visits, surgery, office visits, and prescriptions, all of which are covered under the terms of the plans. As in the previous example, this distribution is only an approximation of the real distribution, which would contain a continuum of expenses. However, it is probably adequate for making a decision among the three plans.

TABLE 10.12 Distribution of Medical Expenses for Insurance Example

Total Medical Expense	Probability
\$200	0.30
\$600	0.50
\$1000	0.15
\$5000	0.03
\$15,000	0.02

The next step is to determine the employee's cost for each plan and each outcome. For example, suppose that the employee chooses plan 1 and incurs \$600 in expenses. Then the total cost is the cost of the insurance plus the full amount of the first \$500 in expenses plus 10% of the last \$100 in expenses, that is,

$$24(12) + 500 + 0.1(100) = \$798$$

However, if this employee's medical expenses are only \$200, then the cost is

$$24(12) + 200 = \$488$$

[3] We assume that these terms apply only to the employee; that is, these are not family plans.

The costs for the other plans and other outcomes can be calculated in a similar manner. We list all of the costs in Table 10.13.

The choice is certainly not clear from this table. The plan with the lowest premium, plan 2, looks good if the year's medical expenses are low. This is also true for the no-deductible plan, plan 3, although its cost is quite large in case of a disaster. For moderate medical expenses, plan 1 is obviously inferior, but it is the best for guarding against a disaster. These trade-offs could be illustrated by risk profiles, which you might want to examine. Instead, we turn directly to the decision tree.

TABLE 10.13 Employee Cost Table for Insurance Example

Medical Expense	Plan 1	Plan 2	Plan 3
$200	$488	$212	$300
$600	$798	$612	$420
$1000	$838	$1012	$540
$5000	$1238	$1412	$1740
$15,000	$2238	$2412	$4740

USING PRECISIONTREE

The decision tree can be formed with the following steps.

❶ **Inputs.** Enter the inputs for the three plans and the probabilities from Table 10.12 in the top left portion of the spreadsheet (down to row 15). (See Figure 10.21 and the file MEDICAL.XLS.)

❷ **Cost table.** For later use in the decision tree, calculate the costs to the employee (not counting insurance premiums) in the range B19:D23. To do this, enter the formula

=IF($A19<=B$6,$A19,B$6+B$7*($A19-B$6))

FIGURE 10.21
Inputs and Cost Table for Medical Example

	A	B	C	D
1	**Medical insurance problem**			
2				
3	Inputs for plans			
4		Plan1	Plan2	Plan3
5	Monthly cost	$24	$1	$20
6	Deductible	$500	$1,000	$0
7	Copay Pct	10%	10%	30%
8				
9	Distribution of medical expenses			
10		Expense	Prob	
11		$200	0.3	
12		$600	0.5	
13		$1,000	0.15	
14		$5,000	0.03	
15		$15,000	0.02	
16				
17	Out of pocket cost table (plan along top, expense along side), not including premiums			
18		Plan1	Plan2	Plan3
19	$200	$200	$200	$60
20	$600	$510	$600	$180
21	$1,000	$550	$1,000	$300
22	$5,000	$950	$1,400	$1,500
23	$15,000	$1,950	$2,400	$4,500

in cell B19 and copy this to the range B19:D23. This IF function says that if the medical expense is less than the deductible, the employee pays it all. Otherwise, the employee pays the deductible amount plus a percentage of the remainder.

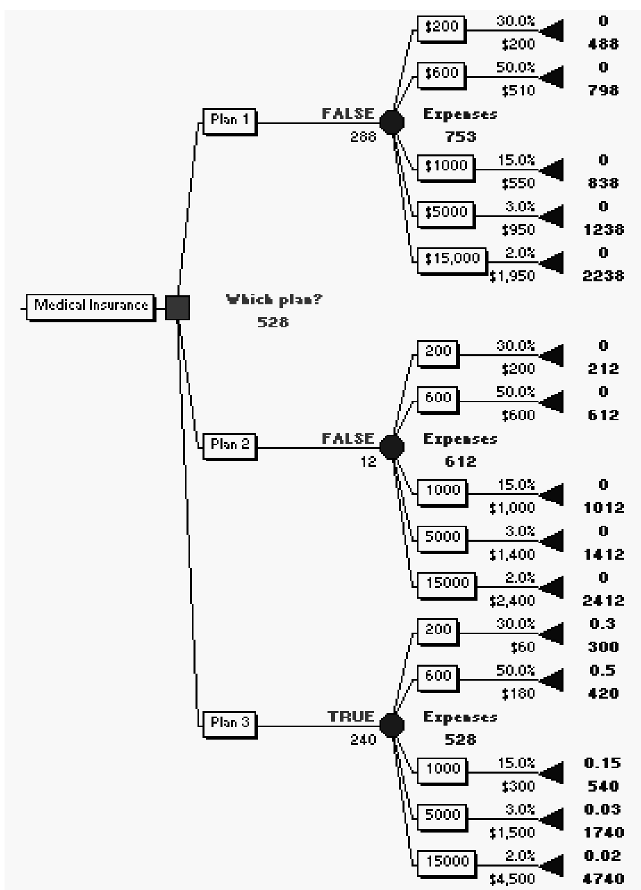
❸ **Decision tree.** Use PrecisionTree to create the decision tree shown in Figure 10.22. Here are some tips. First, create the decision node and decision branches, and enter formulas for their values as 12 times the relevant monthly premiums. Then create a single probability node and its branches, label the branches, and enter formulas for the probabilities with *absolute* references. For example, enter the formula

=$C$11

for the probability of the top branch. Next, copy the probability node to the end nodes below it. (Do you see the effect of the absolute references?) Finally, link the values for all of the probability branches to the cells in the cost table. (We know of no quick way to do this. We entered 15 separate formulas, one for each branch. However, it is much easier to create a cost table and link branch formulas to it than to create the branch formulas directly from input values.)

❹ **Minimize costs.** If we quit here, we would mistakenly choose the *worst* of the three plans. This is because PrecisionTree *maximizes* EMV by default, and in this problem we want to *minimize* the EMV of the costs. However, this is simple to change. Click on the name box at the far left in the decision tree. This brings up a dialog box (not shown here) where we can select the Minimize option.

FIGURE 10.22
Decision Tree for Medical Insurance Example



As we see from Figure 10.22, the optimal plan is plan 3. Its EMV—an expected *cost*—is \$528. The EMVs for plans 1 and 2 are \$753 and \$612. Evidently, this employee's chances of large medical expenses where plan 3 is at its worst are not large enough to outweigh plan 3's no-deductible benefit. However, we might want to experiment with various inputs, either the properties of the plans or the employee's medical expense distribution, to see whether plan 3 continues to be the preferred plan. For example, if the probabilities in Table 10.12 change to 0.30, 0.40, 0.15, 0.10, and 0.05, so that large expenses are much more probable, the EMVs for the three plans become \$827, \$722, and \$750. Now plan 2 is preferred, although the difference in EMV between plans 2 and 3 is quite small.

We can use this insurance example to illustrate one *nonmonetary* aspect of decision problems that is difficult to incorporate into a decision tree. At the university where we teach, there is another insurance plan in addition to the types in the example. Its premiums are low, and there are *no* copayments—the insurer pays all medical expenses. This plan is clearly the cheapest of all plans offered, but it is not chosen by many employees. Why? The plan is through an HMO, where all employees must go to a specified set of physicians; otherwise, the plan does not pay their expenses. Evidently, many employees believe that the "cost" of having to go to physicians they would not choose otherwise outweighs the dollar savings from the plan. ■

The following example illustrates one method for using a *continuous* probability distribution in a decision tree model.

EXAMPLE 10.3

PURCHASING LIGHTBULBS AT FRESHWAY SUPERMARKETS

FreshWay, a chain of supermarkets, requires 24,000 fluorescent lightbulbs for its stores. There are two suppliers of these lightbulbs. Supplies A offers them at \$4.00 per bulb and will replace the first 900 defective bulbs with guaranteed good ones for \$3.00 each. It will replace all defectives after the first 900 for nothing. Supplier B is similar. It will charge \$4.15 per bulb, replace the first 1200 defectives for \$1.00 each, and replace all defectives after the first 1200 for nothing. FreshWay plans to sell these lightbulbs for \$4.40 apiece and charge its customers nothing for replacement of defectives. The only uncertainty is the number of defective bulbs from either supplier. Based on historical data from each supplier, FreshWay believes that the percentage of defectives is normally distributed with mean 4% and standard deviation 1% from supplier A, and mean 4.2% and standard deviation 1.2% from supplier B. Which supplier should be chosen to maximize FreshWay's EMV?

Solution

Let p be the percentage of lightbulbs that are defective. Then the profit to FreshWay from buying from supplier A is

$$\text{Profit} = \begin{cases} 24{,}000(4.40 - 4.00) - (24{,}000p)(3.00) & \text{if } p \leq 900/24{,}000 \\ 24{,}000(4.40 - 4.00) - (900)(3.00) & \text{if } p > 900/24{,}000 \end{cases}$$

A similar expression holds for supplier B. The only random quantity in this expression is p, which is normally distributed. The question is how we can model the *continuous* distribution of p in a *discrete* decision tree—that is, a tree with a discrete number of probability branches. The method usually used is to approximate the continuous normal distribution by a discrete distribution with a relatively small number, say 5, of equally likely values.

The idea is to divide the normal distribution into an equal number of equal probability regions and take the midpoint (in a probability sense) of each region as a value for the decision tree. For example, if we use five points, then each region has probability 0.2. The probability halfway between 0 and 0.2 is 0.1, so the first point on the tree is the 10th percentile of the normal distribution. Similarly, the next point is the 30th percentile, the next is the 50th, the next is the 70th, and the last is the 90th.

Figure 10.23 illustrates the calculations. (See the file LIGHTBULB.XLS.) Through row 13 we enter the given inputs for the problem. Then in rows 17–26 we enter the information we'll use in the decision tree regarding the percentage defective for each supplier. This information is based on the five-point approximation to the normal distribution. For example, the 10th percentile of the normal distribution for supplier A is found in cell C17 with the formula

=NORMINV(B17,$B$12,$C$12)

and this is copied down to cell C21. Then the cost to FreshWay from defectives, assuming the value in C17 is the percentage of defectives, is calculated in cell D17 with the formula

=$C$7*IF(C17<=$D$7/Quantity,Quantity*C17,$D$7)

and it is copied down to cell D21. Similar formulas are used for supplier B.

FIGURE 10.23
Inputs and Calculations for Lightbulb Example

	A	B	C	D
1	**FreshWay lightbulb purchasing example**			
2				
3	Quantity	24000		
4	Selling price	$4.40		
5				
6		UnitCost	ReplaceCost	Charge for first:
7	Supplier A	$4.00	$3.00	900
8	Supplier B	$4.15	$1.00	1200
9				
10	Distribution of percent defective: normal			
11		Mean	Stdev	
12	Supplier A	4.0%	1.0%	
13	Supplier B	4.2%	1.2%	
14				
15	Percentages to use on decision tree			
16		Midpoint probability	Percentile	FreshWay's cost
17	Supplier A	0.1	2.72%	$1,957.28
18		0.3	3.48%	$2,502.43
19		0.5	4.00%	$2,700.00
20		0.7	4.52%	$2,700.00
21		0.9	5.28%	$2,700.00
22	Supplier B	0.1	2.66%	$638.91
23		0.3	3.57%	$856.97
24		0.5	4.20%	$1,008.00
25		0.7	4.83%	$1,159.03
26		0.9	5.74%	$1,200.00

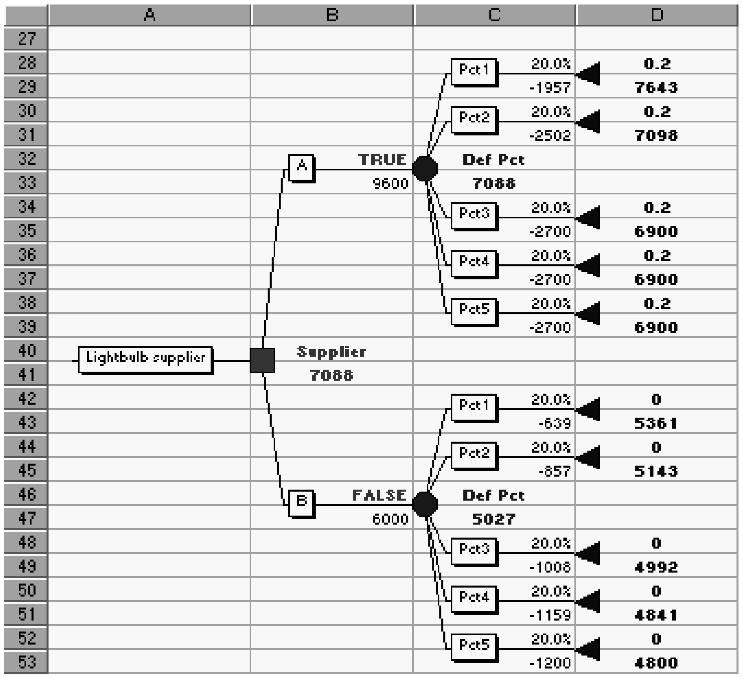
Range names
Quantity: B3
SellingPrice: B4

USING PRECISIONTREE

It is now straightforward to construct the decision tree shown in Figure 10.24 (page 520). We enter the revenue from selling the bulbs and the cost of purchasing them in cells B33 and B47. For example, the formula in cell B33 is

=Quantity*(SellingPrice-B7)

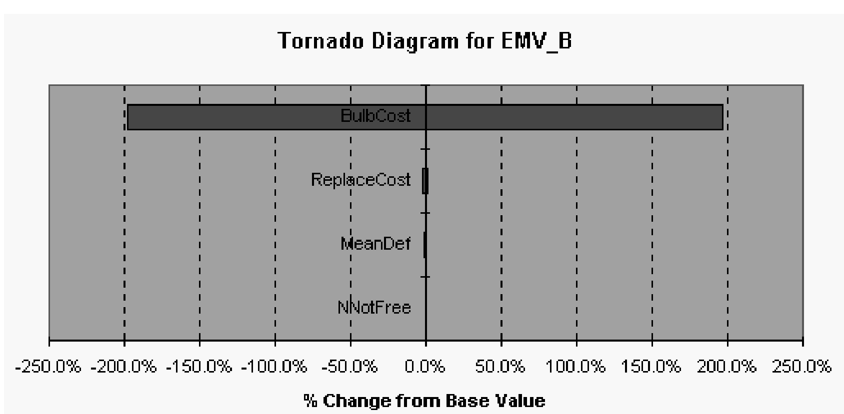
FIGURE 10.24
Decision Tree for Lightbulb Example



Then we link the monetary values below the probability branches to the relevant cells in the D17:D26 range.

The EMVs for suppliers A and B are $7088 and $5027, so supplier A is the clear choice. Evidently, the higher price charged by supplier B and its slightly higher mean percentage of defects outweigh its better deal on replacing defectives. Of course, if supplier B really wants to get FreshWay's business, it could attempt to sweeten its deal in a number of ways. Sensitivity analysis is useful to see how the EMV for supplier B (in cell C47) is affected by the various input parameters. We tried this, varying the inputs in cells B8, C8, D8, and B13 by PrecisionTree's default values (10% in either direction) and keeping track of the change in the EMV for supplier B. The tornado chart in Figure 10.25 makes it very clear that the most important input is the unit purchase cost. The effects of the other three inputs are practically negligible in comparison. If supplier B wants FreshWay's business, it will have to lower its unit purchase cost.

FIGURE 10.25
Tornado Chart to Analyze the EMV for Supplier B



Modeling Issues

The discrete approximation used in Example 10.3 can be used in *any* decision tree with continuous probability distributions, regardless of whether they are normal. We first need to decide how many values to have in the discrete approximation. The usual choices are 5 or 3. (Surprisingly, a three-point approximation does an adequate job in many situations.) Then we need to use the "inverse" function—in the previous example it was the NORMINV function—to find the values to use in the decision tree. The appropriate inverse function is available in Excel for a number of widely used continuous distributions. ■

PROBLEMS

Skill-Building Problems

10. Each day the manager of a local bookstore must decide how many copies of the community newspaper to order for sale in her shop. She must pay the newspaper's publisher $0.40 for each copy and sells the newspapers to local residents for $0.50 each. Newspapers that are unsold at the end of day are considered worthless. The probability distribution of the number of copies of the newspaper purchased daily at her shop is provided in Table 10.14. Employ a decision tree to find the bookstore manager's profit-maximizing daily order quantity.

Table 10.14 Distribution of Daily Local Newspaper Demand

Daily Demand for Local Newspaper	Probability
10	0.10
11	0.15
12	0.30
13	0.20
14	0.15
15	0.10

11. Two construction companies are bidding against one another for the right to construct a new community center building in Lewisburg, Pennsylvania. The first construction company, Fine Line Homes, believes that its competitor, Buffalo Valley Construction, will place a bid for this project according to the distribution shown in Table 10.15. Furthermore, Fine Line Homes estimates that it will cost $160,000 for its own company to construct this building. Given its fine reputation and long-standing service within the local community, Fine Line Homes believes that it will likely be awarded the project in the event that it and Buffalo Valley Construction submit exactly the same bids. Employ a decision tree to identify Fine Line Homes' profit-maximizing bid for the new community center building.

Table 10.15 Distribution of Possible Competing Bids for Construction Project

Buffalo Valley Construction's Bid	Probability
$160,000	0.40
$165,000	0.30
$170,000	0.20
$175,000	0.10

12. Suppose that you have sued your employer for damages suffered when you recently slipped and fell on an icy surface that should have been treated by your company's physical plant department. Specifically, your injury resulting from this accident was sufficiently serious that you, in consultation with your attorney, decided to sue your company for $500,000. Your company's insurance provider has offered to settle this suit with you out of court. If you decide to reject the settlement and go to court, your attorney is confident that you will win the case but is uncertain about the amount the court will award you in damages. He has provided his assessment of the probability distribution of the court's award to you in Table 10.16 (page 522). Let S be the insurance provider's proposed out-of-court settlement (in dollars). For which values of S will you decide to accept the settlement? For which values of S will you choose to take your chances in court? Of course, you are seeking to maximize the expected payoff from this litigation.

TABLE 10.16 Distribution of Possible Court Award Amounts

Amount of Court Award	Probability
$0	0.025
$50,000	0.075
$100,000	0.100
$200,000	0.125
$300,000	0.175
$400,000	0.200
$500,000	0.300

13. Suppose that one of your colleagues has $2000 available to invest. Assume that all of this money must be placed in one of three investments: a particular money market fund, a stock, or gold. Each dollar your colleague invests in the money market fund earns a virtually guaranteed 12% annual return. Each dollar he invests in the stock earns an annual return characterized by the probability distribution provided in Table 10.17. Finally, each dollar he invests in gold earns an annual return characterized by the probability distribution given in Table 10.18.

a. If your colleague must place all of his available funds in a single investment, which investment should he choose to maximize his expected earnings over the next year?

TABLE 10.17 Distribution of Annual Returns for Given Stock

Annual Returns for Given Stock	Probability
0%	0.10
6%	0.20
12%	0.40
18%	0.20
24%	0.10

TABLE 10.18 Distribution of Annual Returns for Gold

Annual Returns for Gold	Probability
−36%	0.10
−12%	0.20
12%	0.40
36%	0.20
60%	0.10

b. Suppose now that your colleague can place all of his available funds in one of these three investments as before, or he can invest $1000 in one alternative and $1000 in another. Assuming that he seeks to maximize his expected total earnings in one year, how should he allocate his $2000?

Skill-Extending Problems

14. A home appliance company is interested in marketing an innovative new product. The company must decide whether to manufacture this product essentially on its own or employ a subcontractor to manufacture it. Table 10.19 contains the estimated probability distribution of the cost of manufacturing 1 unit of this new product (in dollars) under the alternative that the home appliance company produces the item on its own. Table 10.20 contains the estimated probability distribution of the cost of purchasing 1 unit of this new product (in dollars) under the alternative that the home appliance company commissions a subcontractor to produce the item.

a. Assuming that the home appliance company seeks to minimize the expected unit cost of manufacturing or buying the new product, should the company make the new product or buy it from a subcontractor?

b. Perform sensitivity analysis on the optimal expected cost. Under what conditions, if any,

TABLE 10.19 Distribution of Unit Production Cost under "Make" Alternative

Cost Per Unit	Probability
$50	0.20
$53	0.25
$55	0.30
$57	0.20
$59	0.05

TABLE 10.20 Distribution of Unit Production Cost under "Buy" Alternative

Cost Per Unit	Probability
$50	0.10
$53	0.20
$55	0.40
$57	0.20
$59	0.10

would the home appliance company select an alternative different from the one you identified in part **a**?

15. A grapefruit farmer in central Florida is trying to decide whether to take protective action to limit damage to his crop in the event that the overnight temperature falls to a level well below freezing. He is concerned that if the temperature falls sufficiently low and he fails to make an effort to protect his grapefruit trees, he runs the risk of losing his entire crop, which is worth approximately \$75,000. Based on the latest forecast issued by the National Weather Service, the farmer estimates that there is a 60% chance that he will lose his entire crop if it is left unprotected. Alternatively, the farmer can insulate his fruit by spraying water on all of the trees in his orchards. This action, which would likely cost the farmer C dollars, would prevent total devastation but might not completely protect the grapefruit trees from incurring some damage as a result of the unusually cold overnight temperatures. Table 10.21 contains the assessed distribution of possible damages (in dollars) to the insulated fruit in light of the cold weather forecast. Of course, this farmer seeks to minimize the expected total cost of coping with the threatening weather.

TABLE 10.21 Distribution of Damages to Insulated Grapefruit Crop

Damage to Grapefruit Crop	Probability
\$0	0.30
\$5000	0.15
\$10,000	0.10
\$15,000	0.15
\$20,000	0.30

a. Find the maximum value of C below which the farmer will choose to insulate his crop in hopes of limiting damage as result of the unusually cold weather.

b. Set C equal to the value identified in part **a**. Perform sensitivity analysis to determine under what conditions, if any, the farmer might be better off not spraying his grapefruit trees and taking his chances in spite of the threat to his crop.

16. Consider again the department store buyer's decision problem described in Problem 6. Assume now that consumer demand for the new tennis shoes model (in hundreds of pairs) during the upcoming summer season is *normally* distributed with mean 6 and standard deviation 1.5.

a. Formulate a payoff table that specifies the contribution to profit (in dollars) from the sale of the tennis shoes by this department store chain for each possible purchase decision (in hundreds of pairs) and each outcome with respect to consumer demand. Use an appropriate discrete approximation of the given normal demand distribution.

b. Construct a decision tree to identify the buyer's course of action that maximizes the expected profit (in dollars) earned by the department store chain from the purchase and subsequent sale of tennis shoes in the coming year.

17. Consider again the purchasing agent's decision problem described in Problem 9. Assume now that the proportion of defective components supplied by this supplier is well described by the *triangular* distribution with parameters 0, 0, and 1. (This is called the **right triangular** distribution with range 1.)

a. Formulate a payoff table that specifies the microcomputer manufacturer's total cost (in dollars) of purchasing and repairing (if necessary) a complete lot of components for each possible decision and each outcome with respect to the proportion of defective items. Use an appropriate discrete approximation of the given triangular distribution for the proportion of defective items.

b. Construct a decision tree to identify the purchasing agent's course of action that minimizes the expected total cost (in dollars) of achieving a complete lot of satisfactory components.

18. A retired partner from Goldman Sachs has 1 million dollars available to invest in particular stocks or bonds. Each investment's annual rate of return depends on the state of the economy in the forthcoming year. Table 10.22 (page 524) contains the distribution of returns for these stocks and bonds as a function of the economy's state in the coming year. This investor wants to allocate her \$1 million to maximize her expected total return 1 year from now.

a. If $X = Y = 15\%$, find the optimal investment strategy for this investor.

b. For which values of X (where $10\% < X < 20\%$) and Y (where $12.5\% < Y < 17.5\%$), if any, will this investor prefer to place all of her available funds in the given stocks to maximize her expected total return one year from now?

c. For which values of X (where $10\% < X < 20\%$) and Y (where $12.5\% < Y < 17.5\%$), if any, will this investor prefer to place all of her available funds in the given bonds to maximize her expected total return one year from now?

TABLE 10.22 Distribution of Annual Returns for Given Stocks and Bonds

State of the Economy	Probability	Annual Returns for Given Stocks	Annual Returns for Given Bonds
Very strong	0.20	25%	20%
Moderately strong	0.40	20%	17.5%
Fair	0.25	X%	Y%
Moderately weak	0.10	10%	12.5%
Very weak	0.05	5%	10%

10.4 MULTISTAGE DECISION PROBLEMS

So far, all of the examples have required a single decision. We now examine a problem where the decision maker must make at least two decisions that are separated in time, such as when a company must decide whether to buy information that will help it make a second decision. The following example illustrates the typical situation.

EXAMPLE 10.4

MARKETING A NEW PRODUCT AT ACME

The Acme Company is trying to decide whether to market a new product. As in many new-product situations, there is considerable uncertainty about whether the new product will eventually "catch on." Acme believes that it might be prudent to introduce the product in a regional test market before introducing it nationally. Therefore, the company's first decision is whether to conduct the test market. Acme estimates that the fixed cost of the test market is \$3 million. If it decides to conduct the test market, it must then wait for the results. Based on the results of the test market, it can then decide whether to market the product nationally, in which case it will incur a fixed cost of \$90 million. On the other hand, if the original decision is *not* to run a test market, then the final decision—whether to market the product nationally—can be made without further delay. Acme's unit margin, the difference between its selling price and its unit variable cost, is \$18 (in the test market and in the national market).

Acme classifies the results in either the test market or the national market as great, fair, or awful. Each of these is accompanied by a forecast of total units sold. These sales volumes (in 1000s of units) are 200, 100, and 30 for the test market and 6000, 3000, and 900 for the national market. Based on previous test markets for similar products, Acme estimates that probabilities of the three test market outcomes are 0.3, 0.6, and 0.1. Then, based on historical data from previous products that were test marketed and eventually marketed nationally, it assesses the probabilities of the national market outcomes given each possible test market outcome. If the test market is great, the probabilities for the national market outcomes are 0.8, 0.15, and 0.05. If the test market is fair, these probabilities are 0.3, 0.5, and 0.2. If the test market is awful, they are 0.05, 0.25, and 0.7. (Note how the probabilities of the national market outcomes tend to mirror the test market outcomes.)

The company wants to use a decision tree approach to find the best strategy.

Solution

We begin by discussing the three basic elements of this decision problem: the possible strategies, the possible outcomes and their probabilities, and the value model. The possible strategies are clear. Acme must first decide whether to conduct a test market. Then it must decide whether to introduce the product nationally. However, it is important to realize that if Acme decides to conduct a test market, it can base the national market decision on the results of the test market. In this case its final strategy will be a **contingency plan**, where it conducts the test market, then introduces the product nationally if it receives sufficiently positive test market results and abandons the product if it receives sufficiently negative test market results. The optimal strategies from many multistage decision problems involve similar contingency plans.

Regarding the uncertain outcomes and their probabilities, we note that the given probabilities—probabilities of test market outcomes and *conditional* probabilities of national market outcomes given test market outcomes—are exactly the ones we need in the decision tree. This is because the test market outcome is known *before* the national market outcome will occur. However, suppose Acme decides not to run a test market and then decides to market nationally. Then what are the probabilities of the national market outcomes?

It is important to realize that we cannot simply assess three *new* probabilities for this situation. These probabilities are *implied* by the given probabilities. This follows from the rules of conditional probability. If we let T_1, T_2, and T_3 be the test market outcomes, and N be any of the national market outcomes, then by the addition rule for probability and the conditional probability formula,

$$P(N) = P(N \text{ and } T_1) + P(N \text{ and } T_2) + P(N \text{ and } T_3) \quad \textbf{(10.1)}$$

$$= P(N|T_1)P(T_1) + P(N|T_2)P(T_2) + P(N|T_3)P(T_3) \quad \textbf{(10.2)}$$

(This is sometimes called the **law of total probability**.) For example, if N_1 represents a great national market, then from equation (10.1),

$$P(N_1) = (0.8)(0.3) + (0.3)(0.6) + (0.05)(0.1) = 0.425$$

Similarly, we find that $P(N_2) = 0.37$ and $P(N_3) = 0.205$. These are the probabilities we need to use for the probability branches when no test market is used.

Finally, the monetary values in the tree are straightforward. There are fixed costs of test marketing or marketing nationally, and these are incurred as soon as these "go ahead" decisions are made. From that point, we observe the sales volumes and multiply them by the unit margin to obtain the profits.

Using PrecisionTree

The inputs for the decision tree appear in Figure 10.26 (page 526). (See file ACME.XLS.) The only calculated values in this part of the spreadsheet are in row 28, which follow from equation (10.1). Specifically, the formula in cell B28 is

=SUMPRODUCT(B22:B24,$B$16:$B$18)

which we copy across row 28. The tree is then straightforward to build and label, as shown in Figure 10.27 (page 527). Note how the fixed costs of test marketing and marketing nationally appear on the decision branches where they occur, so that only the selling profits need to be placed on the probability branches. Also, the probabilities on the various probability branches are exactly those listed in Figure 10.26.

The interpretation of this tree is fairly straightforward if we realize that each value just below each node name is an EMV. For example, the 807 in cell B43 is the EMV for

FIGURE 10.26
Inputs for Acme Marketing Example

	A	B	C	D
1	**Acme marketing example**			
2				
3	Fixed costs ($1000s)		**Range names:**	
4	Test mkt	$3,000	NatlMktCost: B5	
5	National mkt	$90,000	TestMktCost: B4	
6			UnitMargin: B18	
7	Unit margin	$18		
8				
9	Possible quantities sold (1000s)			
10		Test mkt	Natl mkt	
11	Great	200	6000	
12	Fair	100	3000	
13	Awful	30	900	
14				
15	Probabilities of test outcomes			
16	Great	0.30		
17	Fair	0.60		
18	Awful	0.10		
19				
20	Probabilities of natl mkt outcomes, given test mkt outcomes			
21		Natl great	Natl fair	Natl awful
22	Test great	0.80	0.15	0.05
23	Test fair	0.30	0.50	0.20
24	Test awful	0.05	0.25	0.70
25				
26	Probabilities of natl mkt outcomes without test mkt (calculated from above inputs)			
27		Natl great	Natl fair	Natl awful
28		0.425	0.370	0.205

the entire decision problem. It means that Acme's best EMV is $807,000. As another example, the 5910 in cell D47 means that if Acme ever gets to that point—the test market has been conducted and it has been great—the EMV for ACME is $5,910,000. Each of these EMVs has been calculated by the folding-back procedure we discussed earlier, starting from the right and working back toward the left. PrecisionTree takes EMVs at probability nodes and maximums at decision nodes.

We can also see Acme's optimal strategy by following the "TRUE" branches from left to right. Acme should first run a test market. If the test market results are great, then the product should be marketed nationally. However, if the test market results are only fair or awful, the product should be abandoned. In these cases the prospects from a national market look bleak, so Acme should cut its losses. (And there *are* losses. In these latter two cases, Acme has spent $3,000,000 on the test market and has recouped only $1,800,000 or $540,000 on test market sales.)

The risk profile from the optimal strategy appears in Figure 10.28 (page 528). It is based on the data in Figure 10.29 (page 528). (These were obtained by clicking on PrecisionTree's "staircase" button and selecting the Statistics and Risk Profile options.) We see that there is a small chance of two possible large losses (approximately $73 million and $35 million), there is a 70% chance of a moderate loss of about $1 or $2 million, and there is a 24% chance of an $18.6 million profit. Of course, the net effect is an EMV of $807,000.

You might argue that the large potential losses and the slightly higher than 70% chance of *some* loss should persuade Acme to abandon the product right away—without a test market. However, this is what "playing the averages" with EMV is all about. Since the EMV of this optimal strategy is greater than 0, the EMV of abandoning the product right away, Acme should go ahead with this optimal strategy if the company is indeed an EMV maximizer. In Section 10.8 we will see how this reasoning can change if Acme is a risk-averse decision maker—as it might be with multimillion dollar losses looming in the future!

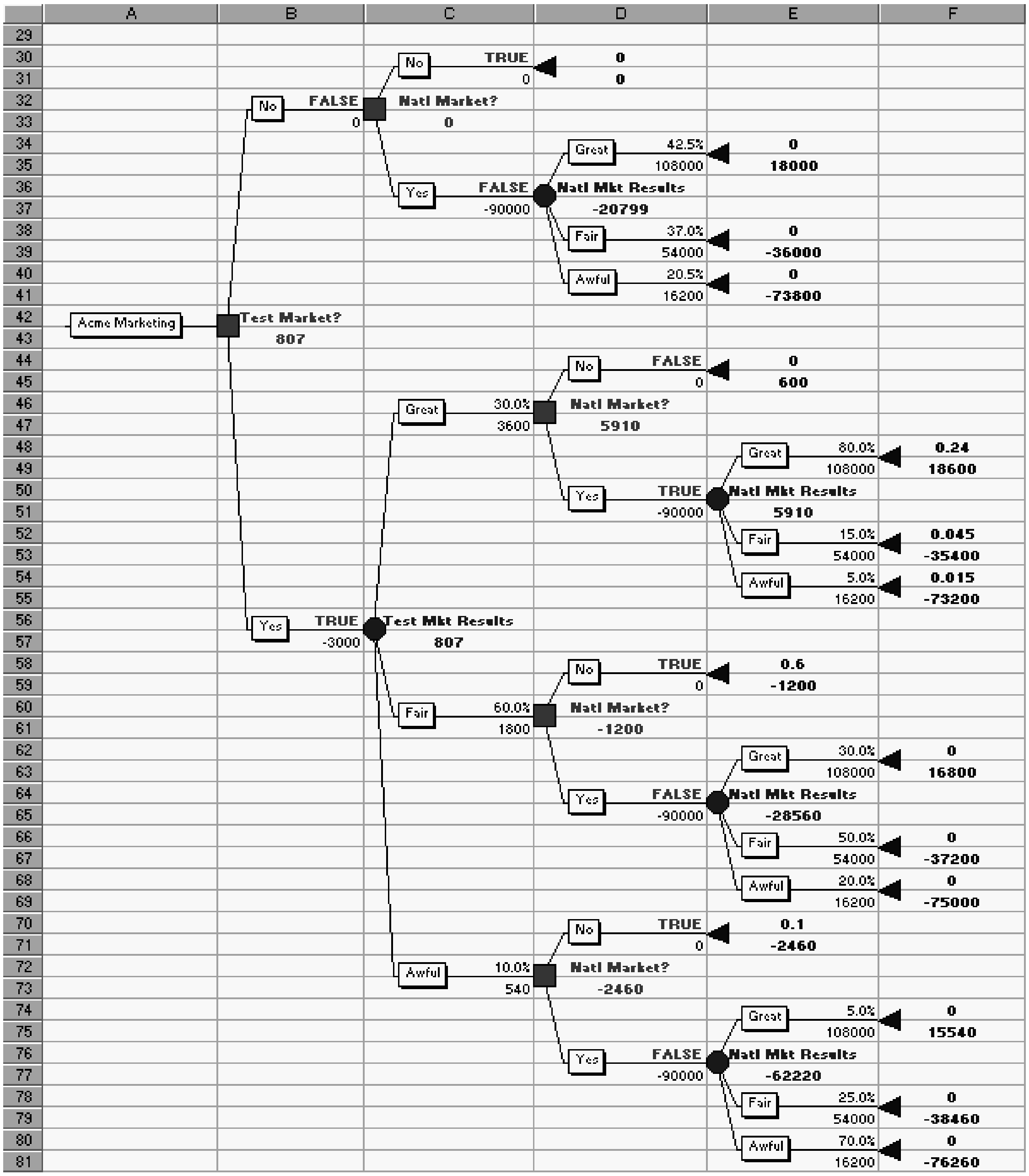


FIGURE 10.27 Decision Tree for Acme Marketing Example

Expected Value of Sample Information The role of the test market in the Acme marketing example is to provide information in the form of more accurate probabilities of national market results. Information usually costs something, as it does in Acme's problem. Currently, the fixed cost of the test market is $3 million, which is evidently not too much to pay because Acme's best strategy is to conduct the test market. However, we might ask how much this test market is worth. This is easy to answer. From the decision tree in Figure 10.27, we see that the EMV from test marketing is $807,000 better than the decision *not* to test market (and then abandon the product). Therefore, if the fixed cost of test marketing were any more than $807,000 above its current value,

FIGURE 10.28
Risk Profile of Optimal Strategy

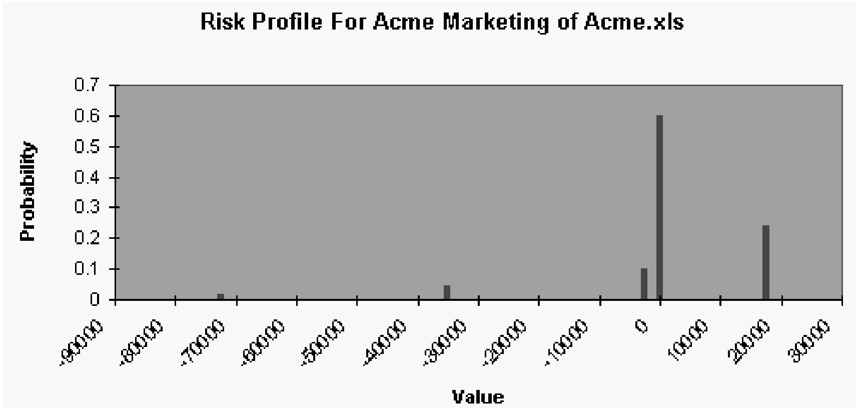


FIGURE 10.29
Distribution of Profit/Loss from the Optimal Strategy

	A	B	C
16	PROFILE:		
17	#	X	P
18	1	-73200	0.015
19	2	-35400	0.045
20	3	-2460	0.1
21	4	-1200	0.6
22	5	18600	0.24

Acme would be better not to run a test market. Equivalently, the most Acme would be willing to pay for the test market (as a fixed cost) is $3.807 million.

This value is called the **expected value of sample information**, or **EVSI**. In general, we can write the following expression for EVSI:

$$\text{EVSI} = \text{EMV with } \textit{free} \text{ information} - \text{EMV without information}$$

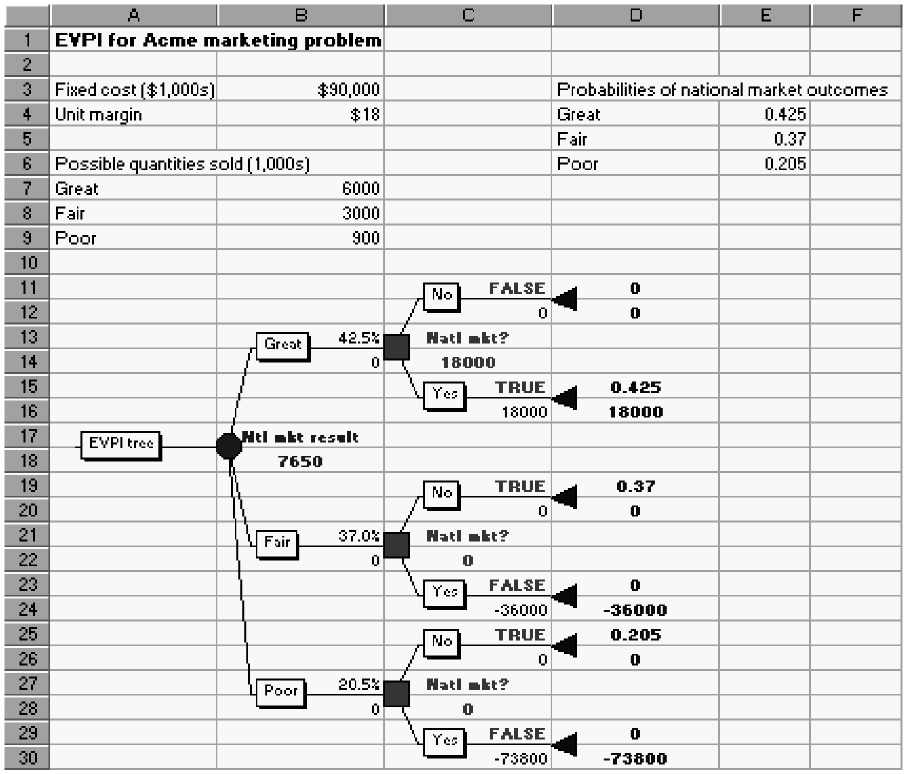
In Acme's problem, the EMV with free information is $3.807 million (just don't charge for the test market fixed cost), and the EMV without any test market information is $0 (because Acme abandons the product when there is no test market available). Therefore,

$$\text{EVSI} = \$3.807 - \$0 = \$3.807 \text{ million}$$

Expected Value of Perfect Information The reason for the term *sample* is that the information does not remove all uncertainty about the future. That is, even after the test market results are in, there is still uncertainty about the national market results. Therefore, we might go one step further and ask how much *perfect* information is worth. We can imagine perfect information as an envelope that contains the true final outcome (of the national market). That is, either "the national market will be great," "the national market will be fair," or "the national market will be awful" is written inside the envelope. Admittedly, no such envelope exists, but if it did, how much would Acme be willing to pay for it?

We can answer this question with the simple decision tree in Figure 10.30. Now the probability node on the left corresponds to opening the envelope. Its probabilities are the same as before (when there is no test market available). Note the reasoning here. Acme doesn't know what the contents of the envelope will be, so we need a probability node. However, once the envelope is opened, the true national market outcome will be revealed. At that point Acme's decision is fairly obvious. If it learns that a national market will be great, it knows the product will be profitable and will market it. Otherwise, if it learns that the national market will be fair or poor, it knows

FIGURE 10.30
EVPI for Acme Marketing Example



that there will be a loss from marketing nationally, so it will abandon the product. Folding back in the usual way produces an EMV of $7.65 million.

Now compare this $7.65 million with the EMV in the top part of Figure 10.27 that results from no test market, namely, $0. The difference, $7.65 million, is called the **expected value of perfect information**, or **EVPI**. It represents the maximum amount the company would pay for perfect information about the final outcome (of the national market). In general, the expression for EVPI is

$$\text{EVPI} = \text{EMV with } \textit{free} \text{ perfect information} - \text{EMV with no information}$$

In Acme's case this expression becomes

$$\text{EVPI} = \$7.65 - \$0 = \$7.65 \text{ million}$$

The EVPI may appear to be an irrelevant concept since perfect information is almost never available—at *any* price. However, it is often useful because it represents an *upper bound* on the EVSI for any potential sample information. That is, no sample information can ever be worth more than the EVPI. For example, if Acme is contemplating an expensive test market with an anticipated fixed cost of more than $8 million, then there is really no point in pursuing it any further. The information gained from this test market, no matter how reliable it is, cannot possibly justify its cost because its cost is greater than the EVPI. ■

PROBLEMS

Skill-Building Problems

19. The senior executives of an oil company are trying to decide whether to drill for oil in a particular field in the Gulf of Mexico. It costs the company $300,000 to drill in the selected field. Company executives believe that if oil is found in this field its estimated value will be $1,800,000. At present, this oil company believes that there is a 50% chance that the selected field actually contains oil. Before drilling, the company can hire a geologist at a cost of $30,000 to prepare a report that contains a recommendation regarding drilling in the selected field. There is a 55% chance that the geologist will issue a favorable recommendation and a 45% chance that the geologist will issue an unfavorable recommendation. Given a favorable recommendation from the geologist, there is a 75% chance that the field actually contains oil. Given an unfavorable recommendation from the geologist, there is a 15% chance that the field actually contains oil.

- **a.** Assuming that this oil company wishes to maximize its expected net earnings, determine its optimal strategy through the use of a decision tree.
- **b.** Compute and interpret the expected value of sample information (EVSI) in this decision problem.
- **c.** Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

20. A local certified public accountant must decide which of two copying machines to purchase for her expanding business. The cost of purchasing the first machine is $4500, and the cost of maintaining the first machine each year is uncertain. The CPA's office manager believes that the annual maintenance cost for the first machine will be $0, $150, or $300 with probabilities 0.35, 0.35, and 0.30, respectively. The cost of purchasing the second machine is $3000, and the cost of maintaining the second machine through a guaranteed maintenance agreement is $225 per year.

Before the purchase decision is made, the CPA can hire an experienced copying machine repairperson to evaluate the quality of the first machine. Such an evaluation would cost the CPA $60. If the repairperson believes that the first machine is satisfactory, there is a 65% chance that its annual maintenance cost will be $0 and a 35% chance that its annual maintenance cost will be $150. If, however, the repairperson believes that the first machine is unsatisfactory, there is a 60% chance that its annual maintenance cost will be $150 and a 40% chance that its annual maintenance cost will be $300. The CPA's office manager believes that the repairperson will issue a satisfactory report on the first machine with probability 0.50.

- **a.** Provided that the CPA wishes to minimize the expected total cost of purchasing and maintaining one of these two machines for a 1-year period, which machine should she purchase? When, if ever, would it be worthwhile for the CPA to obtain the repairperson's review of the first machine?
- **b.** Compute and interpret the expected value of sample information (EVSI) in this decision problem.
- **c.** Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

21. FineHair is developing a new product to promote hair growth in cases of male pattern baldness. If FineHair markets the new product and it is successful, the company will earn $500,000 in additional profit. If the marketing of this new product proves to be unsuccessful, the company will lose $350,000 in development and marketing costs. In the past, similar products have been successful 60% of the time. At a cost of $50,000, the effectiveness of the new restoration product can be thoroughly tested. If the results of such testing are favorable, there is an 80% chance that the marketing efforts of this new product will be successful. If the results of such testing are not favorable, there is a mere 30% chance that the marketing efforts of this new product will be successful. FineHair currently believes that the probability of receiving favorable test results is 0.60.

- **a.** Identify the strategy that maximizes FineHair's expected net earnings in this situation.
- **b.** Compute and interpret the expected value of sample information (EVSI) in this decision problem.
- **c.** Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

22. Hank is considering placing a bet on the upcoming showdown between the Penn State and Michigan football teams in State College. The winner of this contest will represent the Big Ten Conference in the Rose Bowl on New Year's Day. Without any additional information, Hank believes that each team has an equal chance of winning this big game. If he wins the bet, he will win $500; if he loses the bet, he will lose $550. Before placing his bet, he may decide to pay his friend Al, who happens to be a football sportswriter for the *Philadelphia Enquirer*, $50 for Al's expert prediction on the game. Assume that Al predicts that Penn State will win similar games 55% of the time, and that Michigan will win similar games 45% of the time. Furthermore, Hank knows that when Al predicts that Penn State will win, there is a 70% chance that Penn State will indeed win the football game. Finally, when Al predicts that Michigan will win, there is a 20% chance that Penn State will proceed to win the upcoming game.

a. In order to maximize his expected profit from this betting opportunity, how should Hank proceed in this situation?

b. Compute and interpret the expected value of sample information (EVSI) in this decision problem.

c. Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

23. A product manager at Clean & Brite seeks to determine whether her company should market a new brand of toothpaste. If this new product succeeds in the marketplace, C&B estimates that it could earn $1,800,000 in future profits from the sale of the new toothpaste. If this new product fails, however, the company expects that it could lose approximately $750,000. If C&B chooses not to market this new brand, the product manager believes that there would be little, if any, impact on the profits earned through sales of C&B's other products. The manager has estimated that the new toothpaste brand will succeed with probability 0.55. Before making her decision regarding this toothpaste product, the manager can spend $75,000 on a market research study. Such a study of consumer preferences will yield either a positive recommendation with probability 0.50 or a negative recommendation with probability 0.50. Given a positive recommendation to market the new product, the new brand will eventually succeed in the marketplace with probability 0.75. Given a negative recommendation regarding the marketing of the new product, the new brand will eventually succeed in the marketplace with probability 0.25.

a. In order to maximize expected profit in this case, what course of action should the C&B product manager take?

b. Compute and interpret the expected value of sample information (EVSI) in this decision problem.

c. Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

Skill-Extending Problems

24. A publishing company is trying to decide whether to publish a new business law textbook. Based on a careful reading of the latest draft of the manuscript, the publisher's senior editor in the business textbook division assesses the distribution of possible payoffs earned by publishing this new book. Table 10.23 contains this probability distribution. Before making a final decision regarding the publication of the book, the editor can learn more about the text's potential for success by thoroughly surveying business law instructors teaching at universities across the country. Historical frequencies based on similar surveys administered in the past are provided in Table 10.24.

a. Find the strategy that maximizes the publisher's expected payoff (in dollars).

TABLE 10.23 Distribution of Payoffs for New Business Law Textbook

Textbook Performance	Probability	Estimated Payoff (if published)
Very strong	0.20	$100,000
Moderately strong	0.20	$50,000
Fair	0.20	$0
Poor	0.40	−$50,000

TABLE 10.24 Historical Frequencies of Combinations of Past Survey Results and Actual Outcomes

Survey Indication/ Actual Performance	Very Strong	Moderately Strong	Fair	Poor
Very strong	13	12	2	3
Moderately strong	10	20	6	4
Fair	5	12	15	8
Poor	1	3	9	22

b. What is the most (in dollars) that the publisher should be willing to pay to conduct a new survey of business law instructors?

c. If the actual cost of conducting the given survey is less than the amount identified in part **a**, what should the publisher do?

d. Assuming that a survey could be constructed that provides "perfect information" to the publisher, how much should the company be willing to pay to acquire and implement such a survey?

25. Sharp Outfits is trying to decide whether to ship some customer orders now via UPS or wait until after the threat of another UPS strike is over. If Sharp Outfits decides to ship the requested merchandise now and the UPS strike takes place, the company will incur $60,000 in delay and shipping costs. If Sharp Outfits decides to ship the customer orders via UPS and no strike occurs, the company will incur $4000 in shipping costs. If Sharp Outfits decides to postpone shipping its customer orders via UPS, the company will incur $10,000 in delay costs regardless of whether or not UPS goes on strike. Let p represent the probability that UPS will go on strike and impact Sharp Outfits's shipments.

a. For which values of p, if any, does Sharp Outfits minimize its expected total cost by choosing to postpone shipping its customer orders via UPS?

b. Suppose now that, at a cost of $1000, Sharp Outfits can purchase information regarding the likelihood of a UPS strike in the near future. Based on similar strike threats in the past, the probability that this information indicates the occurrence of a UPS strike is 27.5%. If the purchased information indicates the occurrence of a UPS strike, the chance of a strike actually occurring is $0.105/0.275$. If the purchased information does not indicate the occurrence of a UPS strike, the chance of a strike actually occurring is $0.680/0.725$. Provided that $p = 0.15$, what strategy should Sharp Outfits pursue to minimize its expected total cost?

c. Continuing part **b**, compute and interpret the expected value of sample information (EVSI) when $p = 0.15$.

d. Continuing part **b**, compute and interpret the expected value of perfect information (EVPI) when $p = 0.15$.

10.5 BAYES' RULE

In multistage decision problems we typically have alternating sets of decision nodes and probability nodes. The decision maker makes a decision, some uncertain outcomes are observed, the decision maker makes another decision, more uncertain outcomes are observed, and so on. In the resulting decision tree, all probability branches at the *right* of the tree are conditional on outcomes that have occurred earlier, to their left. Therefore, the probabilities on these branches are of the form $P(A|B)$, where B is an event that occurs *before* event A in time. However, it is sometimes more natural to assess conditional probabilities in the opposite order, that is, $P(B|A)$. Whenever this is the case, we require **Bayes' rule** to obtain the probabilities we need on the tree. Essentially, Bayes' rule is a mechanism for updating probabilities as new information becomes available. We illustrate the mechanics of Bayes' rule in the following example. [See Feinstein (1990) for a real application of this example.]

EXAMPLE 10.5

DRUG TESTING COLLEGE ATHLETES

If an athlete is tested for a certain type of drug usage (steroids, say), then the test result will be either positive or negative. However, these tests are never perfect. Some athletes who are drug free test positive, and some who are drug users test negative. The former are called false positives; the latter are called false negatives. We will assume that 5%

of all athletes use drugs, 3% of all tests on drug-free athletes yield false positives, and 7% of all tests on drug users yield false negatives. The question then is what we can conclude from a positive or negative test result.

Solution

Let D and ND denote that a randomly chosen athlete is or is not a drug user, and let $T+$ and $T-$ indicate a positive or negative test result. We are given the following probabilities. First, since 5% of all athletes are drug users, we know that $P(D) = 0.05$ and $P(ND) = 0.95$. These are called **prior** probabilities because they represent the chance that an athlete is or is not a drug user *prior* to the results of a drug test. Second, from the information on drug test accuracy, we know the conditional probabilities $P(T+|ND) = 0.03$ and $P(T-|D) = 0.07$. But a drug-free athlete tests either positive or negative, and the same is true for a drug user. Therefore, we also have the probabilities $P(T-|ND) = 0.97$ and $P(T+|D) = 0.93$. These four conditional probabilities of test results given drug user status are often called the **likelihoods** of the test results.

Given these priors and likelihoods, we want **posterior** probabilities such as $P(D|T+)$, the probability that an athlete who tested positive is a drug user, or $P(ND|T-)$, the probability that an athlete who tested negative is drug free. They are called posterior probabilities because they are assessed *after* the drug test results. This is where Bayes' rule enters. We will develop Bayes' rule in some generality and then apply it to the present example.

Let A be any "information" event, such as the result of a drug test, and let $B_1, B_2, \ldots, B_n$ be any mutually exclusive and exhaustive set of events. That is, exactly one of the B_i's must occur. To apply Bayes' rule, we assume that the prior probabilities $P(B_1), P(B_2), \ldots, P(B_n)$ are given, as are the likelihoods $P(A|B_i)$ for each i. Then we want the posterior probabilities $P(B_i|A)$ for each i. Bayes' rule shows how to find these. For any i, we have

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A|B_1)P(B_1) + \cdots + P(A|B_n)P(B_n)}$$ **Bayes' rule**

This formula says that a typical posterior probability is a ratio. The numerator is a likelihood times a prior, and the denominator is the sum of likelihoods times priors.

Before illustrating Bayes' rule numerically, we make two other observations about the terms in Bayes' rule. First, we can use the multiplication rule of probability to write any product of a likelihood and a prior as

$$P(A|B_i)P(B_i) = P(A \text{ and } B_i)$$

The probability on the right, that *both* A and B_i occur, is called a **joint** probability. Second, we can use the definition of conditional probability directly to write

$$P(B_i|A) = \frac{P(A \text{ and } B_i)}{P(A)}$$

Therefore, the probability in the denominator of Bayes' rule is really just the probability of A:

$$P(A) = P(A|B_1)P(B_1) + \cdots + P(A|B_n)P(B_n)$$

As we will see shortly, this natural by-product of Bayes' rule will come in very handy in decision trees.

It is fairly easy to implement Bayes' rule in a spreadsheet, as illustrated in Figure 10.31 for the drug example. Here A corresponds to either test result, and B_1 and B_2 correspond to D and ND. (See the file DRUGBAYES.XLS.[4]) In words, we want to see how the chances of D and ND change after seeing the results of the drug test.

The given priors and likelihoods are listed in the ranges B5:C5 and B9:C10. We then calculate the products of likelihoods and priors in the range B15:C16. The formula in cell B15 is

=B$5*B9

and this is copied to the rest of the B15:C16 range. Their row sums are calculated in the range D15:D16. These represent the unconditional probabilities of the two possible results. They are also (as we saw above) the denominators of Bayes' rule. Finally, we calculate the posterior probabilities in the range B21:C22. The formula in cell B21 is

=B15/$D15

and this is copied to the rest of the B21:C22 range. The various 1's in the margins of Figure 10.31 are row sums or column sums that must equal 1. We show them only as checks of our logic.

FIGURE 10.31
Bayes' Rule for Drug-Testing Example

	A	B	C	D
1	**Illustration of Bayes' rule using drug example**			
2				
3	Prior probabilities of drug user status			
4		User	Non-user	
5		0.05	0.95	1
6				
7	Likelihoods of test results, given drug user status			
8		User	Non-user	
9	Test positive	0.93	0.03	
10	Test negative	0.07	0.97	
11		1	1	
12				
13	Joint probabilities of drug user status and test results			
14		User	Non-user	Unconditional
15	Test positive	0.0465	0.0285	0.075
16	Test negative	0.0035	0.9215	0.925
17				1
18				
19	Posterior probabilities of drug user status			
20		User	Non-user	
21	Test positive	0.620	0.380	1
22	Test negative	0.004	0.996	1

Note that a negative test result leaves little doubt that the athlete is drug free. The posterior probability that the athlete is drug free, given a negative test result, is 0.996. However, there is still a lot of doubt about an athlete who tests positive. The posterior probability that the athlete uses drugs, given a positive test result, is only 0.620. This asymmetry occurs because of the prior probabilities. We are fairly certain that a randomly selected athlete is drug free because only 5% of all athletes use drugs. It takes a lot of evidence to convince us otherwise. This initial bias, plus the fact that the test produces a few false positives, means that athletes with positive test results still have a decent chance (probability 0.380) of being drug free. Is this a valid argument

[4]The Bayes2 sheet in this file illustrates how Bayes' rule can be used when there are more than two possible test results and/or drug user categories.

for not requiring drug testing of athletes? We explore this question in the following continuation of the drug-testing example. It all depends on the “costs.” (It might also depend on whether there is a second type of test that could help confirm the findings of the first test. However, we won’t consider such a test.) ■

EXAMPLE 10.5 (CONTINUED)

DRUG TESTING COLLEGE ATHLETES

The administrators at State University are trying to decide whether to institute mandatory drug testing for the athletes. They have the same information about priors and likelihoods as in the previous example, but now they want to use a decision tree approach to see whether the benefits outweigh the costs.[5]

Solution

We have already discussed the uncertain outcomes and their probabilities. Now we need to discuss the decision alternatives and the monetary values—the other two elements of a decision analysis. We will assume that there are only two alternatives: perform drug testing on all athletes or don’t perform any drug testing. In the former case we assume that if an athlete tests positive, this athlete is barred from sports.

The “monetary” values are more difficult to assess. They include

- the benefit B from correctly identifying a drug user and barring him or her from sports
- the cost C_1 of the test itself for a single athlete (materials and labor)
- the cost C_2 of falsely accusing a nonuser (and barring him or her from sports)
- the cost C_3 of not identifying a drug user (either by not testing at all or by obtaining a false negative)
- the cost C_4 of violating a nonuser’s privacy by performing the test

It is clear that only C_1 is a direct monetary cost that is easy to measure. However, the other “costs” and the benefit B are real, and they must be compared on some scale to enable administrators to make a rational decision. We will do so by comparing everything to the cost C_1, to which we will assign value 1. (This does not mean that the cost of testing an athlete is necessarily \$1; it just means that we will express all other costs as multiples of C_1.) Clearly, there is a lot of subjectivity involved in making these comparisons, so sensitivity analysis on the final decision tree is a must.

Before developing this decision tree, it is useful to form a benefit–cost table for both alternatives and all possible outcomes. Because we will eventually maximize expected net *benefit*, all benefits in this table have a positive sign and all costs have a negative sign. These net benefits appear in Table 10.25 (page 536). The first two columns are relevant if no tests are performed; the last four are relevant when testing is performed. For example, if a positive test is obtained for a nonuser, there are three

[5]Again, see Feinstein (1990) for an enlightening discussion of this drug-testing problem at a real university.

TABLE 10.25 Net Benefit for Drug-Testing Example

Don't Test		Perform Test			
D	ND	D and $T+$	ND and $T+$	D and $T-$	ND and $T-$
$-C_3$	0	$B - C_1$	$-(C_1 + C_2 + C_4)$	$-(C_1 + C_3)$	$-(C_1 + C_4)$

costs: the cost of the test (C_1), the cost of falsely accusing the athlete (C_2), and the cost of violating the nonuser's privacy (C_4). The other entries are obtained similarly.

The solution with PrecisionTree shown in Figure 10.32 is now fairly straightforward. (See the file DRUG.XLS.) We first enter all of the benefits and costs in an input section. These, together with the Bayes' rule calculations from before, appear at the top of the spreadsheet. Then we use PrecisionTree in the usual way to build the tree and enter the links to the values and probabilities.

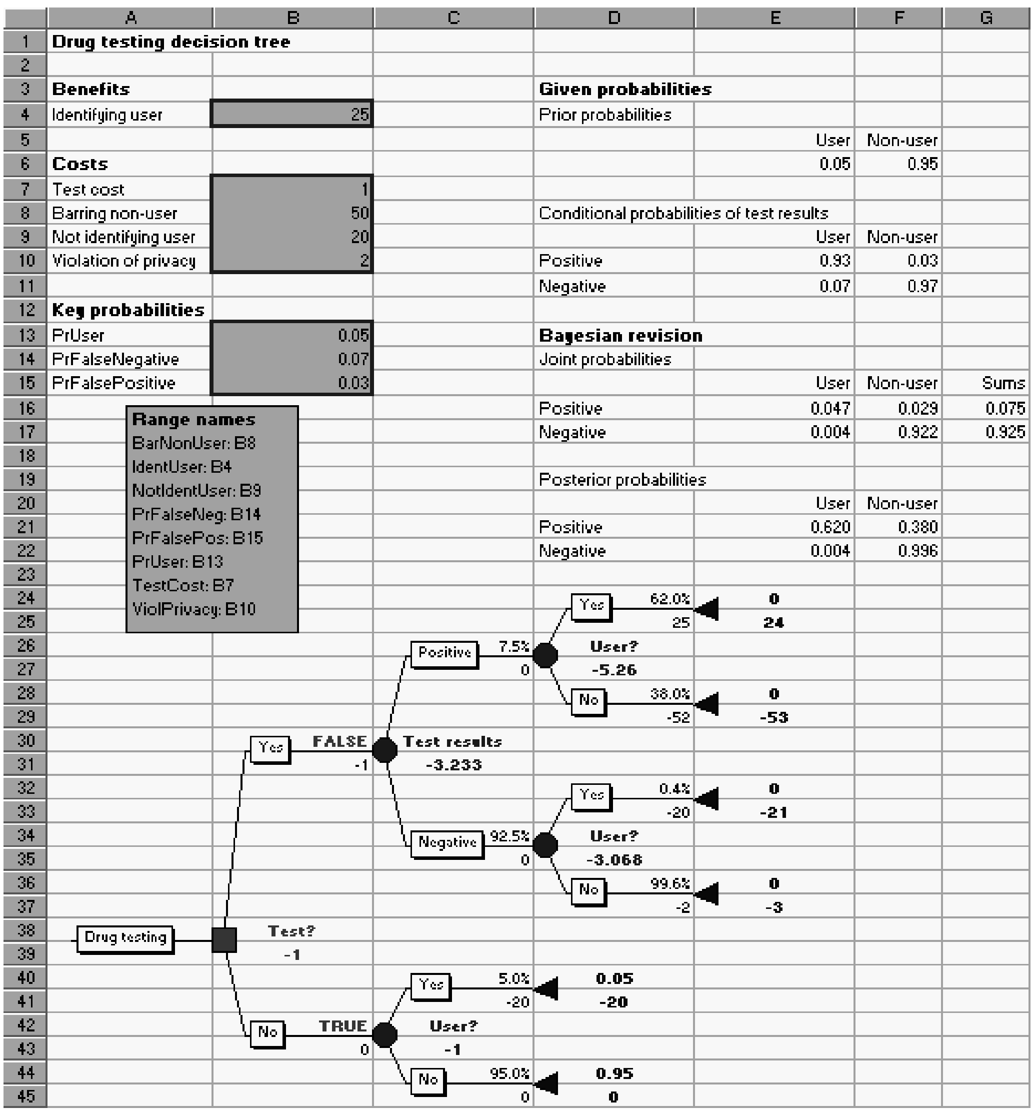


FIGURE 10.32 Decision Tree for Drug-Testing Example

Before we interpret this solution, we discuss the timing (from left to right). If drug testing is performed, the result of the drug test is observed first (a probability node). Each test result leads to an action (bar from sports or don't), and then the eventual benefit or cost depends on whether the athlete uses drugs (again a probability node). You might argue that the university never knows for certain whether the athlete uses drugs, but we must include this information in the tree to get the benefits and costs correct. If no drug testing is performed, then there is no intermediate test result node or branches.

Now to the interpretation. First, we discuss the benefits and costs shown in Figure 10.32. These were chosen fairly arbitrarily, but with some hope of reflecting reality. They say that the largest cost is falsely accusing (and barring) a nonuser. This is 50 times as large as the cost of the test. The benefit of identifying a drug user is only half this large, and the cost of not identifying a user is 40% as large as barring a nonuser. The violation of privacy of a nonuser is twice as large as the cost of the test. Based on these values, the decision tree implies that drug testing should *not* be performed. The EMVs for testing and for not testing are both negative, indicating that the costs outweigh the benefits for each, but the EMV for not testing is slightly *less* negative.[6]

What would it take to change this decision? We'll start with the assumption, probably accepted by most people in our society, that the cost of falsely accusing a nonuser (C_2) ought to be the largest of the benefits or costs in the range B4:B10. In fact, because of possible legal costs, we might argue that C_2 should be *more* than 50 times the cost of the test. But if we increase C_2, the scales are tipped even farther in the direction of not testing. On the other hand, if the benefit B from identifying a user and/or the cost C_3 for not identifying a user increase, then testing might be the preferred alternative. We tried this, keeping C_2 constant at 50. When B and C_3 both had value 45, no testing was still optimal, but when they both increased to 50—the same magnitude as C_2—then testing won out by a small margin. However, it would be difficult to argue that B and C_3 should be of the same magnitude as C_2.

Other than the benefits and costs, the only other thing we might vary is the accuracy of the test, measured by the error probabilities in cells B14 and B15. Presumably, if the test makes fewer false positives and false negatives, testing might be a more attractive alternative. We tried this, keeping the benefits and costs the same as those shown in Figure 10.32 but changing the error probabilities. Even when each error probability was decreased to 0.01, however, the no-testing alternative was still optimal—by a fairly wide margin.

In summary, based on a number of reasonable assumptions and parameter settings, this example has shown that it is difficult to make a case for mandatory drug testing. ■

[6]The university in the Feinstein (1990) study came to the same conclusion.

PROBLEMS

Skill-Building Problems

26. Consider a population of 2000 individuals, 800 of whom are women. Assume that 300 of the women in this population earn at least $60,000 per year, and 200 of the men earn at least $60,000 per year.

- **a.** What is the probability that a randomly selected individual from this population earns less than $60,000 per year?
- **b.** If a randomly selected individual is observed to earn less than $60,000 per year, what is the probability that this person is a man?
- **c.** If a randomly selected individual is observed to earn at least $60,000 per year, what is the probability that this person is a woman?

27. Yearly automobile inspections are required for residents of the state of Pennsylvania. Suppose that 18% of all inspected cars in Pennsylvania have problems that need to be corrected. Unfortunately, Pennsylvania state inspections fail to detect these problems 12% of the time. Consider a car that is inspected and is found to be free of problems. What is the probability that there is indeed something wrong that the inspection has failed to uncover?

28. Consider again the landowner's decision problem described in Problem 3. Suppose now that, at a cost of $90,000, the landowner can request that a soundings test be performed on the site where natural gas is believed to be present. The company that conducts the soundings concedes that 30% of the time the test will indicate that no gas is present when it actually is. When natural gas is not present in a particular site, the soundings test is accurate 90% of the time.

- **a.** Given that the landowner pays for the soundings test and the test indicates that gas is present, what is the landowner's revised estimate of the probability of finding gas on this site?
- **b.** Given that the landowner pays for the soundings test and the test indicates that gas is not present, what is the landowner's revised estimate of the probability of not finding gas on this site?
- **c.** Should the landowner request the given soundings test at a cost of $90,000? Explain why or why not. If not, when (if ever) would the landowner choose to obtain the soundings test?

29. The chief executive officer of a firm in a highly competitive industry believes that one of her key employees is providing confidential information to the competition. She is 90% certain that this informer is the vice-president of finance, whose contacts have been extremely valuable in obtaining financing for the company. If she decides to fire this vice-president and he is the informer, she estimates that the company will gain $500,000. If she decides to fire this vice-president but he is not the informer, the company will lose his expertise and still have an informer within the staff; the CEO estimates that this outcome would cost her company about $2.5 million. If she decides not to fire this vice-president, she estimates that the firm will lose $1.5 million whether or not he actually is the informer (since in either case the informer is still with the company).

Before deciding whether to fire the vice-president for finance, the CEO could order lie detector tests. To avoid possible lawsuits, the lie detector tests would have to be administered to all company employees, at a total cost of $150,000. Another problem she must consider is that the available lie detector tests are not perfectly reliable. In particular, if a person is lying, the test will reveal that the person is lying 95% of the time. Moreover, if a person is not lying, the test will indicate that the person is not lying 85% of the time.

- **a.** In order to minimize the expected total cost of managing this difficult situation, what strategy should the CEO adopt?
- **b.** Should the CEO order the lie detector tests for all of her employees? Explain why or why not.
- **c.** Determine the maximum amount of money that the CEO should be willing to pay to administer lie detector tests.

30. A customer has approached a bank for a $10,000 one-year loan at a 12% interest rate. If the bank does not approve this loan application, the $10,000 will be invested in bonds that earn a 6% annual return. Without additional information, the bank believes that there is a 4% chance that this customer will default on the loan, assuming that the loan is approved. If the customer defaults on the loan, the bank will lose $10,000.

At a cost of $100, the bank can thoroughly investigate the customer's credit record and supply a favorable or unfavorable recommendation. Past experience indicates that in cases where the customer did not default on the approved loan, the probability of receiving a favorable recommendation on the basis of the credit investigation was 77/96. Furthermore, in cases where the customer defaulted on the approved loan, the probability of receiving a favorable recommendation on the basis of the credit investigation was 1/4.

- **a.** What course of action should the bank take to maximize its expected profit?
- **b.** Compute and interpret the expected value of sample information (EVSI) in this decision problem.

c. Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

31. A company is considering whether to market a new product. Assume, for simplicity, that if this product is marketed, there are only two possible outcomes: success or failure. The company assesses that the probabilities of these two outcomes are p and $1 - p$, respectively. If the product is marketed and it proves to be a failure, the company will lose \$450,000. If the product is marketed and it proves to be a success, the company will gain \$750,000. Choosing not to market the product results in no gain or loss for the company.

The company is also considering whether to survey prospective buyers of this new product. The results of the consumer survey can be classified as favorable, neutral, or unfavorable. In similar cases where proposed products proved to be market successes, the likelihoods that the survey results were favorable, neutral, and unfavorable were 0.6, 0.3, and 0.1, respectively. In similar cases where proposed products proved to be market failures, the likelihoods that the survey results were favorable, neutral, and unfavorable were 0.1, 0.2, and 0.7, respectively. The total cost of administering this survey is C dollars.

a. Let $p = 0.4$. For which values of C, if any, would this company choose to conduct the consumer survey?

b. Let $p = 0.4$. What is the largest amount that this company would be willing to pay for perfect information about the potential success or failure of the new product?

c. Let $p = 0.5$ and $C = \$15{,}000$. Find the strategy that maximizes the company's expected earnings in this situation. Does the optimal strategy involve conducting the consumer survey? Explain why or why not.

32. The U.S. government is attempting to determine whether immigrants should be tested for a contagious disease. Let's assume that the decision will be made on a financial basis. Furthermore, assume that each immigrant who is allowed to enter the United States and has the disease costs the country \$100,000. Also, each immigrant who is allowed to enter the United States and does not have the disease will contribute \$10,000 to the national economy. Finally, assume that x percent of all potential immigrants have the disease. The U.S. government can choose to admit all immigrants, admit no immigrants, or test immigrants for the disease before determining whether they should be admitted. It costs T dollars to test a person for the disease; the test result is either positive or negative. A person who does not have the disease *always* tests negative. However, 20% of all people who *do* have the disease test negative. The government's goal is to maximize the expected net financial benefits per potential immigrant.

a. Let $x = 10$ (i.e., 10%). What is the largest value of T at which the U.S. government will choose to test potential immigrants for the disease?

b. How does your answer to the question in part **a** change when x increases to 15?

c. Let $x = 10$ and $T = \$100$. Find the government's optimal strategy in this case.

d. Let $x = 10$ and $T = \$100$. Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

Skill-Extending Problems

33. A city in Ohio is considering replacing its fleet of gasoline-powered automobiles with electric cars. The manufacturer of the electric cars claims that this municipality will experience significant cost savings over the life of the fleet if it chooses to pursue the conversion. If the manufacturer is correct, the city will save about \$1.5 million dollars. If the new technology employed within the electric cars is faulty, as some critics suggest, the conversion to electric cars will cost the city \$675,000. A third possibility is that less serious problems will arise and the city will break even with the conversion. A consultant hired by the city estimates that the probabilities of these three outcomes are 0.30, 0.30, and 0.40, respectively.

The city has an opportunity to implement a pilot program that would indicate the potential cost or savings resulting from a switch to electric cars. The pilot program involves renting a small number of electric cars for 3 months and running them under typical conditions. This program would cost the city \$75,000. The city's consultant believes that the results of the pilot program would be significant but not conclusive; she submits Table 10.26 (page 398), a compilation of probabilities based on the experience of other cities, to support her contention. For example, the first row of her table indicates that given that a conversion to electric cars actually results in a savings of \$1.5 million, the conditional probabilities that the pilot program will indicate that the city saves money, loses money, and breaks even are 0.6, 0.1, and 0.3, respectively.

a. What actions should this city take to maximize the expected savings?

b. Should the city implement the pilot program at a cost of \$75,000?

c. Compute and interpret the expected value of sample information (EVSI) in this decision problem.

TABLE 10.26 Likelihoods of Pilot Program Outcomes Given Actual Conversion Outcomes

Actual Outcome of Conversion/ Pilot Program Indication	Savings	Loss	Break Even
Savings	0.6	0.1	0.3
Loss	0.1	0.4	0.5
Break Even	0.4	0.2	0.4

34. A manufacturer must decide whether to extend credit to a retailer who would like to open an account with the firm. Past experience with new accounts indicates that 45% are high-risk customers, 35% are moderate-risk customers, and 20% are low-risk customers. If credit is extended, the manufacturer can expect to lose \$60,000 with a high-risk customer, make \$50,000 with a moderate-risk customer, and make \$100,000 with a low-risk customer. If the manufacturer decides not to extend credit to a customer, the manufacturer neither makes nor loses any money.

Prior to making a credit extension decision, the manufacturer can obtain a credit rating report on the retailer at a cost of \$2000. The credit agency concedes that its rating procedure is not completely reliable. In particular, the credit rating procedure will rate a low-risk customer as a moderate-risk customer with probability 0.10 and as a high-risk customer with probability 0.05. Furthermore, the given rating procedure will rate a moderate-risk customer as a low-risk customer with probability 0.06 and as a high-risk customer with probability 0.07. Finally, the rating procedure will rate a high-risk customer as a low-risk customer with probability 0.01 and as a moderate-risk customer with probability 0.05.

a. Find the strategy that maximizes the manufacturer's expected net earnings.

b. Should the manufacturer routinely obtain credit rating reports on those retailers who seek credit approval? Why or why not?

c. Compute and interpret the expected value of sample information (EVSI) in this decision problem.

35. A television network earns an average of \$1.6 million each season from a hit program and loses an average of \$400,000 each season on a program that turns out to be a flop. Of all programs picked up by this network in recent years, 25% turn out to be hits and 75% turn out to be flops. At a cost of C dollars, a market research firm will analyze a pilot episode of a prospective program and issue a report predicting whether the given program will end up being a hit. If the program is actually going to be a hit, there is a 90% chance that the market researchers will predict the program to be a hit. If the program is actually going to be a flop, there is a 20% chance that the market researchers will predict the program to be a hit.

a. Assuming that $C = \$160{,}000$, identify the strategy that maximizes this television network's expected profit in responding to a newly proposed television program.

b. What is the maximum value of C that this television network should be willing to incur in choosing to hire the market research firm?

c. Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

10.6 INCORPORATING ATTITUDES TOWARD RISK

Rational decision makers are sometimes willing to violate the EMV maximization criterion when large amounts of money are at stake. These decision makers are willing to sacrifice some EMV to reduce risk. Are you ever willing to do so personally? Consider the following scenarios.

1. You have a chance to enter a lottery where you will win \$100,000 with probability 0.1 or win nothing with probability 0.9. Alternatively, you can receive \$5000 for certain. How many of you—truthfully—would take the certain \$5000, even though

the EMV of the lottery is $10,000? Or change the $100,000 to $1,000,000 and the $5000 to $50,000 and ask yourself whether you'd prefer the sure $50,000!

2. You can either buy collision insurance on your expensive new car or not buy it, where the insurance costs a certain premium and carries some deductible provision. If you decide to pay the premium, then you are essentially paying a certain amount to avoid a gamble—the possibility of wrecking your car and not having it insured. You can be sure that the premium is greater than the expected cost of damage; otherwise, the insurance company would not stay in business. Therefore, from an EMV standpoint you should not purchase the insurance. But how many of you drive without this type of insurance?

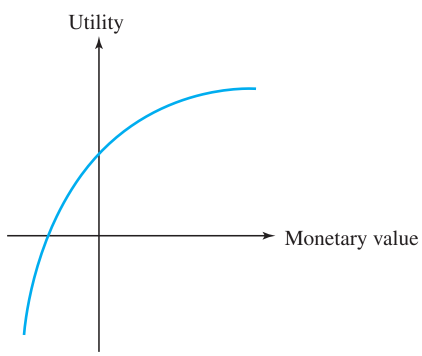
These examples, the second of which is certainly realistic, illustrate situations where rational people do not behave as EMV maximizers. Then how do they act? This question has been studied extensively by many researchers, both mathematically and behaviorally. Although the answer is still not agreed upon universally, most researchers believe that if certain basic behavioral assumptions hold, people are **expected utility** maximizers—that is, they choose the alternative with the largest expected utility. Although we will not go deeply into the subject of expected utility maximization, the discussion in this section will acquaint you with the main ideas.

Utility Functions

We begin by discussing an individual's **utility function**. This is a mathematical function that transforms monetary values—payoffs and costs—into **utility values**. Essentially, an individual's utility function specifies the individual's preferences for various monetary payoffs and costs and, in doing so, it automatically encodes the individual's attitudes toward risk. Most individuals are **risk averse**, which means intuitively that they are willing to sacrifice some EMV to avoid risky gambles. In terms of the utility function, this means that every extra dollar of payoff is worth slightly less to the individual than the previous dollar, and every extra dollar of cost is considered slightly more costly (in terms of utility) than the previous dollar. The resulting utility functions are shaped as shown in Figure 10.33. Mathematically, these functions are said to be **increasing** and **concave**. The increasing part means that they go uphill—everyone prefers more money to less money. The concave part means that they increase at a decreasing rate. This is the risk-averse behavior.

There are two problems involved in implementing utility maximization in a real decision analysis. The first is obtaining an individual's (or company's) utility function; we will discuss this below. The second is using the resulting utility function to find the best decision. This second step is actually quite straightforward. We simply substitute

FIGURE 10.33
Risk-Averse Utility Function



utility values for monetary values in the decision tree and then fold back as usual. That is, we calculate expected *utilities* at probability branches and take maximums (of expected utilities) at decision branches. We will look at a numerical example later in this section. So the real work involves finding an individual's (or company's) utility function in the first place.

Assessing a Utility Function

We will outline a method that can be used to estimate a person's utility function. There are two things we must understand about this method. First, it asks the person to make a series of trade-offs. Because each of us has different attitudes toward risk, we will not all make the trade-offs in the same way. Therefore, each of us will obtain our own utility function. Second, even a particular person's utility function is not unique. If $U(x)$ represents a person's utility function, then it turns out that $aU(x) + b$ also describes that person's utility function, for any constants a and b with $a > 0$. They are equivalent in the sense that they lead to exactly the same decisions.

We take advantage of this nonuniqueness by specifying two points on the utility function. Specifically, we begin by asking the person for two monetary values that represent the worst possible loss and the best possible gain imaginable. Let's say these values are $-A$ and B. Then we *arbitrarily* assign utility values 0 and 1 to these two monetary values, that is, $U(-A) = 0$ and $U(B) = 1$. Don't worry about the absolute magnitudes, 0 and 1, we've assigned—we could assign any other values, such as 14 and 320. The important thing is to use these as "anchors" and then obtain other utility values in terms of them.

The procedure is as follows. Given any two known utility values, say, $U(x)$ and $U(y)$, where x and y are monetary values, we present the person with a choice between the following two options:

- Option 1: Obtain a certain payoff of z.
- Option 2: Obtain a payoff of either x or y, depending on the flip of a fair coin.

Then we ask the person to select the monetary value z in option 1 so that he or she is *indifferent* between the two options. If the person is indifferent, then the expected utilities from the two options must be equal. We will call the resulting value of z the **indifference value**. This leads to the equation for $U(z)$:

$$U(z) = 0.5U(x) + 0.5U(y) \quad \textbf{(10.3)}$$

In words, we have generated a new utility value from two known utility values. This process continues until we have enough utility values to approximate a utility curve. (Note that if any of x, y, and z are negative, then "payoff" really means "cost.") We will illustrate this procedure with the following example.

EXAMPLE 10.6

ASSESSING THE UTILITY FUNCTION FOR A SMALL BUSINESS

John Jacobs owns his own business. Because he is about to make an important decision where large losses or large gains are at stake, he wants to use the expected utility criterion to make his decision. He knows that he must first assess his own utility function, so he hires a decision analysis expert, Susan Schilling, to help him out. How might the session between John and Susan proceed?

Solution

Susan first asks John for the largest loss and largest gain he can imagine. He answers with the values \$200,000 and \$300,000, so she assigns utility values $U(-200{,}000) = 0$ and $U(300{,}000) = 1$ as anchors for the utility function. Now she presents John with the choice between two options:

- Option 1: Obtain a payoff of z (really a loss if z is negative).
- Option 2: Obtain a loss of \$200,000 or a payoff of \$300,000, depending on the flip of a fair coin.

Susan reminds John that the EMV of option 2 is \$50,000 (halfway between $-$\$200,000 and \$300,000). He realizes this, but because he is quite risk averse, he would far rather have \$50,000 for certain than take the gamble in option 2. Therefore, the indifference value of z must be less than \$50,000. Susan then poses several values of z to John. Would he rather have \$10,000 for sure or take option 2? He says he'd rather take the \$10,000. Would he rather *pay* \$5000 for sure or take the gamble in option 2? (This is like an insurance premium.) He says he'd rather take option 2. By this time, we know the indifference value of z must be less than \$10,000 and greater than $-$\$5000. With a few more questions of this type, John finally decides on $z = \$5000$ as his indifference value. He is indifferent between obtaining \$5000 for sure and taking the gamble in option 2. We can substitute these values into equation (10.3):

$$U(5000) = 0.5U(-200{,}000) + 0.5U(300{,}000) = 0.5(0) + 0.5(1) = 0.5$$

Note that John is giving up \$45,000 in EMV because of his risk aversion. The EMV of the gamble in option 2 is \$50,000, and he is willing to accept a *sure* \$5000 in its place.

The process would then continue. For example, since she now knows $U(5000)$ and $U(300{,}000)$, Susan could ask John to choose between these options:

- Option 1: Obtain a payoff of z.
- Option 2: Obtain a payoff of \$5000 or a payoff of \$300,000, depending on the flip of a fair coin.

If John decides that his indifference value is now $z = \$130{,}000$, then with equation (10.3) we know that

$$U(130{,}000) = 0.5U(5000) + 0.5U(300{,}000) = 0.5(0.5) + 0.5(1) = 0.75$$

Note that John is now giving up \$22,500 in EMV because the EMV of the gamble in option 2 is \$152,500. By continuing in this manner, Susan can help John assess enough utility values to approximate a continuous utility curve. ■

As this example illustrates, utility assessment is tedious. Even in the best of circumstances, when a trained consultant attempts to assess the utility function of a single person, the process requires the person to make a series of choices between hypothetical alternatives involving uncertain outcomes. Unless the person has some training in probability, these choices can be difficult to understand, let alone make, and it is unlikely that the person will answer *consistently* as the questioning proceeds. The process is even more difficult when a company's utility function is being assessed. Because company executives involved typically have different attitudes toward risk, it is difficult for these people to reach a consensus on a common utility function.

Exponential Utility

For these reasons there are classes of "ready-made" utility functions that have been developed. One important class is called **exponential utility** and has been used in many financial investment analyses. An exponential utility function has only one adjustable numerical parameter, and there are straightforward ways to discover the most appropriate value of this parameter for a particular individual or company. So the advantage of using an exponential utility function is that it is relatively easy to assess. The drawback is that exponential utility functions do not capture all types of attitudes toward risk. Nevertheless, their ease of use has made them popular.

An exponential utility function has the following form:

$$U(x) = 1 - e^{-x/R} \tag{10.4}$$

Here x is a monetary value (a payoff if positive, a cost if negative), $U(x)$ is the utility of this value, and $R > 0$ is an adjustable parameter called the **risk tolerance**. Basically, the risk tolerance measures how much risk the decision maker will tolerate. The larger the value of R, the less risk averse the decision maker is. That is, a person with a large value of R is more willing to take risks than a person with a small value of R.

To assess a person's (or company's) exponential utility function, we need only to assess the value of R. There are a couple of tips for doing this. First, it has been shown that the risk tolerance is approximately equal to that dollar amount R such that the decision maker is indifferent between the following two options:

- Option 1: Obtain no payoff at all.
- Option 2: Obtain a payoff of R dollars or a loss of $R/2$ dollars, depending on the flip of a fair coin.

For example, if you are indifferent between a bet where you win $1000 or lose $500, with probability 0.5 each, and not betting at all, then your R is approximately $1000. From this criterion it certainly makes intuitive sense that a wealthier person (or company) ought to have a larger value of R. This has been found in practice.

A second tip for finding R is based on empirical evidence found by Ronald Howard, a prominent decision analyst. Through his consulting experience with several large companies, he discovered tentative relationships between risk tolerance and several financial variables—net sales, net income, and equity. [See Howard (1992).] Specifically, he found that R was approximately 6.4% of net sales, 124% of net income, and 15.7% of equity for the companies he studied. For example, according to this prescription, a company with net sales of $30 million should have a risk tolerance of approximately $1.92 million. Howard admits that these percentages are only guidelines. However, they do indicate that larger and more profitable companies tend to have larger values of R, which means that they are more willing to take risks involving given dollar amounts.

We illustrate the use of the expected utility criterion, and exponential utility in particular, with the following example.

EXAMPLE 10.7

DECIDING WHETHER TO ENTER RISKY VENTURES AT VENTURE LIMITED

Venture Limited is a company with net sales of $30 million. The company currently must decide whether to enter one of two risky ventures or do nothing. The possible outcomes of the less risky venture are a \$0.5 million loss, a \$0.1 million gain, and a \$1 million gain. The probabilities of these outcomes are 0.25, 0.50, and 0.25. The possible outcomes of the more risky venture are a \$1 million loss, a \$1 million gain, and a \$3 million gain. The probabilities of these outcomes are 0.35, 0.60, and 0.05. If Venture Limited can enter at most one of the two risky ventures, what should it do?

Solution

We will assume that Venture Limited has an exponential utility function. Also, based on Howard's guidelines, we will assume that the company's risk tolerance is 6.4% of its net sales, or \$1.92 million. (We'll do a sensitivity analysis on this parameter later on.) We can substitute into equation (10.4) to find the utility of any monetary outcome. For example, the gain from doing nothing is \$0, and its utility is

$$U(0) = 1 - e^{-0/1.92} = 1 - 1 = 0$$

As another example, the utility of a \$1 million loss is

$$U(-1) = 1 - e^{-(-1)/1.92} = 1 - 1.683 = -0.683$$

These are the values we use (instead of monetary values) in the decision tree.

Using PrecisionTree

Fortunately, PrecisionTree takes care of all the details. After we build a decision tree and label it (with monetary values) in the usual way, we click on the name of the tree (the box on the far left of the tree) to open the dialog box in Figure 10.34. We then fill in the utility function information as shown in the upper right section of the dialog box. This says to use an exponential utility function with risk tolerance 1.92. It also indicates that we want expected utilities (as opposed to EMVs) to appear in the decision tree.

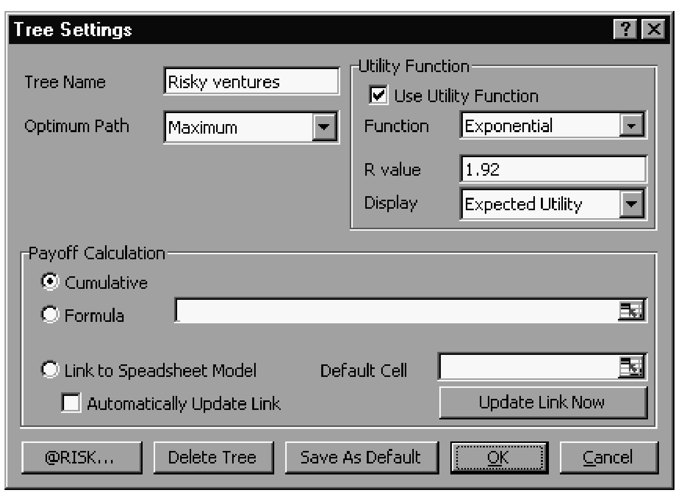
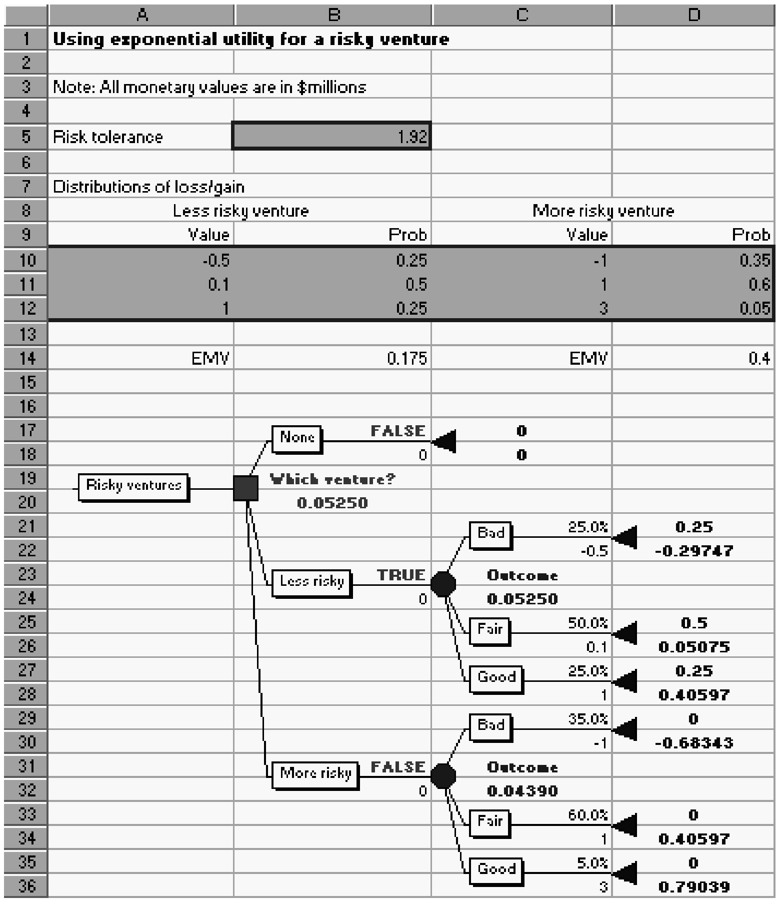


FIGURE 10.34
Dialog Box for Specifying the Exponential Utility Criterion

The completed tree for this example appears in Figure 10.35 (page 546). (See the file VENTURE.XLS.) We build it in exactly the same way as usual and link probabilities and monetary values to its branches in the usual way. For example, there is a link in cell C22 to the monetary value in cell A10. However, the expected values shown in the tree (those shown in color on your screen) are expected *utilities*, and the optimal decision is the one with the largest expected utility. In this case the expected utilities for doing nothing, investing in the less risky venture, and investing in the more

FIGURE 10.35 Decision Tree for Risky Venture Example



risky venture are 0, 0.0525, and 0.0439. Therefore, the optimal decision is to invest in the less risky venture.

Note that the EMVs of the three decisions are \$0, \$0.175 million, and \$0.4 million. The latter two of these are calculated in row 14 as the usual "sumproduct" of monetary values and probabilities. So from an EMV point of view, the more risky venture is definitely best. However, Venture Limited is sufficiently risk averse, and the monetary values are sufficiently large, that the company is willing to sacrifice EMV to reduce its risk.

How sensitive is the optimal decision to the key parameter, the risk tolerance? We can answer this by changing the risk tolerance (through the dialog box in Figure 10.34) and watching how the decision tree changes.[7] You can check that when the company becomes *more* risk tolerant, the more risky venture eventually becomes optimal. In fact, this occurs when the risk tolerance increases to approximately \$2.075 million. In the other direction, when the company becomes *less* risk tolerant, the "do nothing" decision eventually becomes optimal. This occurs when the risk tolerance decreases to approximately \$0.715 million. So the "optimal" decision depends heavily on the attitudes toward risk of Venture Limited's top management.

Certainty Equivalents Now suppose that Venture Limited has only two options. It can either enter the less risky venture or receive a *certain* dollar amount x and avoid

[7]We show the risk tolerance in cell B5, but the values in the decision tree are not linked to that cell. We need to go through the dialog box to change the risk tolerance.

the gamble altogether. We want to find the dollar amount x such that the company is indifferent between these two options. If it enters the risky venture, its expected utility is 0.0525, calculated above. If it receives x dollars for certain, its (expected) utility is

$$U(x) = 1 - e^{-x/1.92}$$

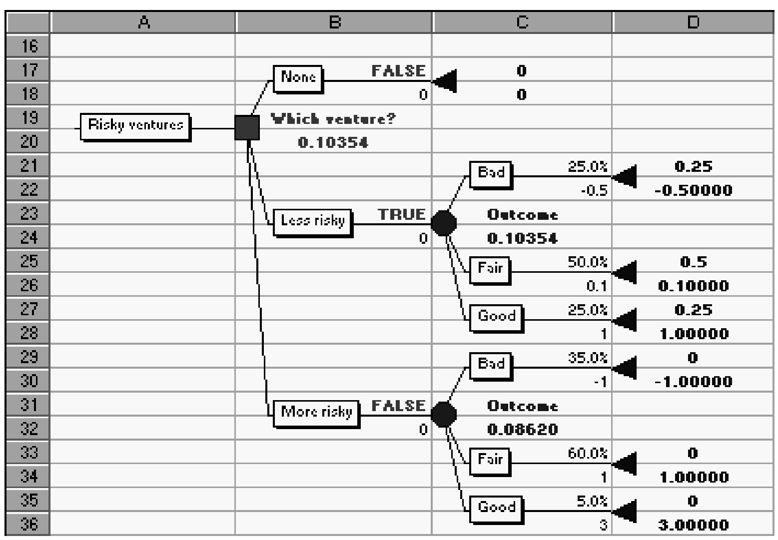
To find the value x where it is indifferent between the two options, we set $1 - e^{-x/1.92}$ equal to 0.0525, or $e^{-x/1.92} = 0.9475$, and solve for x. Taking natural logarithms of both sides and multiplying by -1.92, we obtain

$$x = -1.92 \ln(0.9475) \approx \$0.104 \text{ million}$$

This value is called the **certainty equivalent** of the risky venture. The company is indifferent between entering the less risky venture and receiving \$0.104 million to avoid it. Although the EMV of the less risky venture is \$0.175 million, the company acts as if it is equivalent to a sure \$0.104 million. In this sense, the company is willing to give up the difference in EMV, \$71,000, to avoid a gamble.

By a similar calculation, the certainty equivalent of the more risky venture is approximately \$0.086 million. That is, the company acts as if this more risky venture is equivalent to a sure \$0.086 million, when in fact its EMV is a hefty \$0.4 million! So in this case it is willing to give up the difference in EMV, \$314,000, to avoid this particular gamble. Again, the reason is that the company dislikes risk. We can see these certainty equivalents in PrecisionTree by adjusting the Display box in Figure 10.34 to show Certainty Equivalent. The tree then looks as in Figure 10.36. The certainty equivalents we just discussed appear in cells C24 and C32.

FIGURE 10.36
Decision Tree with Certainty Equivalents



Is Expected Utility Maximization Used?

The above discussion indicates that utility maximization is a fairly involved task. The bottom line, then, is whether the difficulty is worth the trouble. Theoretically, expected utility maximization might be interesting to researchers, but is it really used? The answer appears to be: not very often. For example, one recent article on the practice of decision making [see Kirkwood (1992)] quotes Ronald Howard—the same person we quoted earlier—as having found risk aversion to be of practical concern in only

5% to 10% of business decision analyses. This same article quotes the president of a Fortune 500 company as saying, "Most of the decisions we analyze are for a few million dollars. It is adequate to use expected value (EMV) for these."

With these comments in mind, it is clear that knowledge of expected utility maximization is an important requirement for anyone intending to specialize in the field. In some of the greatest success stories, expected utility maximization was indeed implemented. For nonspecialists, however, a passing knowledge of the concepts is sufficient.

PROBLEMS

Skill-Building Problems

36. Suppose that a decision maker's utility as a function of his wealth, x, is given by $U(x) = \ln x$ (the natural logarithm of x).

- **a.** Is this decision maker risk averse? Explain why or why not.
- **b.** The decision maker now has $10,000 and two possible decisions. For decision 1, he loses $500 for certain. For decision 2, he loses $0 with probability 0.9 and loses $5000 with probability 0.1. Which decision maximizes the expected utility of his net wealth?

37. An investor has $10,000 in assets and can choose between two different investments. If she invests in the first investment opportunity, there is an 80% chance that she will increase her assets by $590,000 and a 20% chance that she will increase her assets by $190,000. If she invests in the second investment opportunity, there is a 50% chance that she will increase her assets by $1.19 million and a 50% chance that she will increase her assets by $1000. This investor has an exponential utility function for final assets with a risk tolerance parameter equal to $600,000. Which investment opportunity will she prefer?

38. Consider again FreshWay's decision problem described in Example 10.3. Suppose now that FreshWay's utility function of profit π, earned from the acquisition and sale of the 24,000 fluorescent lightbulbs, is $U(\pi) = \ln(\pi)$. Find the course of action that maximizes FreshWay's expected utility. How does this optimal decision compare to the optimal decision with an EMV criterion? Explain any difference in the two decisions.

39. Consider again the landowner's decision problem described in Problem 3. Suppose now that the landowner's utility function of financial gain x is $U(x) = x^2$. Find the course of action that maximizes the landowner's expected utility. How does this optimal decision compare to the optimal decision with an EMV criterion? Explain any difference in the two decisions.

40. Consider again Techware's decision problem described in Problem 4. Suppose now that Techware's utility function of net revenue r (measured in dollars), earned from the given marketing opportunities, is $U(r) = 1 - e^{-r/350{,}000}$.

- **a.** Find the course of action that maximizes Techware's expected utility. How does this optimal decision compare to the optimal decision with an EMV criterion? Explain any difference in the two optimal decisions.
- **b.** Repeat part **a** when Techware's utility function is $U(r) = 1 - e^{-r/50{,}000}$.

41. Consider again the bank's customer loan decision problem in Problem 30. Suppose now that the bank's utility function of profit π (in dollars) is $U(\pi) = 1 - e^{-\pi/10{,}000}$. Find the strategy that maximizes the bank's expected utility in this case. How does this optimal strategy compare to the optimal decision with an EMV criterion? Explain any difference in two optimal strategies.

Skill-Extending Problems

42. Suppose that a decision maker has a utility function for monetary gains x given by $U(x) = (x + 10{,}000)^{0.5}$.

- **a.** Show that this decision maker is indifferent between gaining nothing (i.e., $0) and entering a risky situation where she gains $80,000 with probability 1/3 and loses $10,000 with probability 2/3.
- **b.** If there is a 10% chance that one of the decision maker's family heirlooms, valued at $5000, will be stolen during the next year, what is the most that she would be willing to pay each year for an insurance policy that completely covers the potential loss of her cherished item?

43. A decision maker is going to invest $2000 for a period of 6 months. Two potential investments are available to him: U.S. Treasury bills and gold. If this decision maker invests the $2000 in T-bills, he is sure to end the 6-month period with $2592. If this decision maker invests in gold, there is a 75% chance that he will end the 6-month period with

$800 and a 25% chance that he will end up with $20,000. The decision maker's utility function of ending up with x dollars is $U(x) = \sqrt{x}$.

a. Should this decision maker invest in gold or T-bills?

b. Suppose the decision maker invests a proportion y of his $2000 in T-bills and the remaining fraction $(1 - y)$ of his available funds in gold. In this case his gain or loss from either investment is reduced proportionally. For example, if he invests half of his money in gold, he will either lose $600 with probability 0.75 or gain $9000 with probability 0.25. Given the same utility function $U(x) = \sqrt{x}$, find the investor's optimal choice of y.

10.7 CONCLUSION

In this chapter we have discussed methods that can be used in decision-making problems in which future uncertainty is a key element. Perhaps the most important skill we can gain from this chapter is the ability to approach decision problems that include uncertainty in a systematic manner. This systematic approach requires the decision maker to list all possible decisions or strategies, list all possible uncertain outcomes, assess the probabilities of these outcomes (possibly with the aid of Bayes' rule), calculate all necessary monetary values, and finally do the calculations necessary to obtain the best decision. If large dollar amounts are at stake, it might also be necessary to perform a utility analysis, where the decision maker's feelings toward risk are taken into account. Once the basic analysis has been completed, using "best guesses" for the various parameters of the problem, a sensitivity analysis should be conducted to see whether the best decision continues to be best within a range of problem parameters.

PROBLEMS

Skill-Building Problems

44. Ford is going to produce a new vehicle, the Pioneer, and wants to determine the amount of annual capacity it should build. Ford's goal is to maximize the profit from this vehicle over the next 10 years. Each vehicle will sell for $13,000 and incur a variable production cost of $10,000. Building 1 unit of annual capacity will cost $3000. Each unit of capacity will also cost $1000 per year to maintain, even if the capacity is unused. Demand for the Pioneer is unknown but marketing estimates the distribution of annual demand to be as shown in Table 10.27. Assume that unit sales during a year is the minimum of capacity and annual demand.

a. Explain why a capacity of 1,300,000 is not a good choice.

b. Which capacity level should Ford choose?

TABLE 10.27 Distribution of Annual Demand

Annual Demand	Probability
400,000	0.25
900,000	0.50
1,300,000	0.25

45. You are CEO of the venture capital firm D&D. Billy comes to you with an investment proposition. You estimate that your distribution of cash flows from this investment is as shown in Table 10.28.

TABLE 10.28 Distribution of Cash Flow

Cash Flow	Probability
−1,000,000	0.35
500,000	0.60
3,000,000	0.05

a. If you are trying to maximize the expected value of the firm's cash flows, should you take the project?

b. Suppose you assess your firm to be risk averse, with an exponential utility function. You also use the rule of thumb that the firm's risk tolerance is about 6.4% of its annual revenues, which are $30 million. Determine whether D&D should enter the venture.

46. Pizza King (PK) and Noble Greek (NG) are competitive pizza chains. Pizza King believes there is a 25% chance that NG will charge $6 per pizza, a 50% chance NG will charge $8 per pizza, and a 25% chance that NG will charge $10 per pizza. If PK charges price p_1 and NG charges price p_2, PK will sell $100 + 25(p_2 - p_1)$ pizzas. It costs PK $4 to make a pizza. PK is considering charging $5, $6, $7, $8, or $9 per pizza. In order to maximize its expected profit, what price should PK charge for a pizza?

47. Sodaco is considering producing a new product: Chocovan soda. Sodaco estimates that the annual demand for Chocovan, D (in thousands of cases), has the following probability distribution: $P(D = 30) = 0.30$, $P(D = 50) = 0.40$, $P(D = 80) = 0.30$. Each case of Chocovan sells for $5 and incurs a variable cost of $3. It costs $800,000 to build a plant to produce Chocovan. Assume that if $1 is received every year (forever), this is equivalent to receiving $10 at the present time. If Sodaco decides to build the plant and produce Chocovan, find the expected net present value of its profit.

48. Many decision problems have the following simple structure. A decision maker has two possible decisions, 1 and 2. If decision 1 is made, a *sure* cost of c is incurred. If decision 2 is made, there are two possible outcomes, with costs c_1 and c_2 and probabilities p and $1 - p$. We assume that $c_1 < c < c_2$. The idea is that decision 1, the riskless decision, has a "moderate" cost, whereas decision 2, the risky decision, has a "low" cost c_1 or a "high" cost c_2.

a. Find the decision maker's cost table, that is, the cost for each possible decision and each possible outcome.

b. Calculate the expected cost from the risky decision.

c. List as many scenarios as you can think of that have this structure. (Here's an example to get you started. Think of insurance, where you pay a sure premium to avoid a large possible loss.)

49. During the summer, Olympic swimmer Adam Johnson swims every day. On sunny summer days he goes to an outdoor pool, where he may swim for no charge. On rainy days he must go to a domed pool. At the beginning of the summer, he has the option of purchasing a $15 season pass to the domed pool, which allows him use for the entire summer. If he doesn't buy the season pass, he must pay $1 each time he goes there. Past meteorological records indicate that there is a 60% chance that the summer will be sunny (in which case there is an average of 6 rainy days during the summer) and a 40% chance the summer will be rainy (an average of 30 rainy days during the summer).

Before the summer begins, Adam has the option of purchasing a long-range weather forecast for $1. The forecast predicts a sunny summer 80% of the time and a rainy summer 20% of the time. If the forecast predicts a sunny summer, there is a 70% chance that the summer will actually be sunny. If the forecast predicts a rainy summer, there is an 80% chance that the summer will actually be rainy. Assuming that Adam's goal is to minimize his total expected cost for the summer, what should he do? Also find the EVSI and the EVPI.

50. Erica is going to fly to London on August 5, and return home on August 20. It is now July 1. On July 1, she may buy a one-way ticket (for $350) or a round-trip ticket (for $660). She may also wait until August to buy a ticket. On August 1, a one-way ticket will cost $370, and a round-trip ticket will cost $730. It is possible that between July 1 and August 1, her sister (who works for the airline) will be able to obtain a free one-way ticket for Erica. The probability that her sister will obtain the free ticket is 0.30. If Erica has bought a round-trip ticket on July 1 and her sister has obtained a free ticket, she may return "half" of her round trip to the airline. In this case, her total cost will be $330 plus a $50 penalty. Use a decision tree approach to determine how to minimize Erica's expected cost of obtaining round-trip transportation to London.

51. A nuclear power company is deciding whether to build a nuclear power plant at Diablo Canyon or at Roy Rogers City. The cost of building the power plant is $10 million at Diablo and $20 million at Roy Rogers City. If the company builds at Diablo, however, and an earthquake occurs at Diablo during the next 5 years, construction will be terminated and the company will lose $10 million (and will still have to build a power plant at Roy Rogers City). Without further expert information the company believes there is a 20% chance that an earthquake will occur at Diablo during the next 5 years. For $1 million, a geologist can be hired to analyze the fault structure at Diablo Canyon. She will predict either that an earthquake will occur or that an earthquake will not occur. The geologist's past record indicates that she will predict an earthquake on 95% of the occasions for which an earthquake will occur and no earthquake on 90% of the occasions for which an earthquake will not occur. Should the power company hire the geologist? Also find the EVSI and the EVPI.

52. Joan's utility function for her asset position x (for x between 0 and \$160,000) is given by $U(x) = \sqrt{x}/400$.

a. Is Joan risk averse? Explain.

b. Currently, Joan's assets consist of \$10,000 in cash and a \$90,000 home. During a given year, there is a 0.001 probability that Joan's home will be destroyed by fire or other causes. How much should Joan be willing to pay for insurance that covers her home completely from this type of destruction?

53. My current annual income is \$40,000. I believe that I owe \$8000 in taxes. For \$500, I can hire a CPA to review my tax return. There is a 20% chance she will save me \$4000 in taxes and an 80% chance she won't save me anything. If x is my disposable income for the current year, my utility function is given by $U(x) = \sqrt{x}/200$.

a. Am I risk averse or risk seeking?

b. Should I hire the accountant?

Skill-Extending Problems

54. City officials in Ft. Lauderdale, Florida, are trying to decide whether to evacuate coastal residents in anticipation of a major hurricane that may make landfall near their city within the next 48 hours. Based on previous studies, it is estimated that it will cost approximately 1 million dollars to evacuate the residents living along the coast of this major metropolitan area. However, if city officials choose not to evacuate their residents and the storm strikes Fort Lauderdale, there would likely be some deaths as a result of the hurricane's storm surge along the coast. While city officials are reluctant to place an economic value on the loss of human life resulting from such a storm, they realize that it may ultimately be necessary to do so to make a sound judgment in this situation. Prior to making the evacuation decision, city officials consult hurricane experts at the National Hurricane Center in Coral Gables regarding the accuracy of past predictions. They learn that in similar past cases, hurricanes that were *predicted* to make landfall near a particular coastal location actually did so 60% of the time. Moreover, they learn that in past similar cases hurricanes that were predicted *not* to make landfall near a particular coastal location actually did so 20% of the time. Finally, in response to similar threats in the past, weather forecasters have issued predictions of a major hurricane making landfall near a particular coastal location 40% of the time.

a. Let L be the economic valuation of the loss of human life resulting from a coastal strike by the hurricane. Employ a decision tree to help these city officials make a decision that minimizes the expected cost of responding to the threat of the impending storm as a function of L. To proceed, you might begin by choosing an initial value of L and then perform sensitivity analysis on the optimal decision by varying this model parameter. Summarize your findings.

b. For which values of L will these city officials *always* choose to evacuate the coastal residents, regardless of the Hurricane Center's prediction?

55. A homeowner wants to decide whether he should install an electronic heat pump in his home. Given that the cost of installing a new heat pump is fairly large, the homeowner would like to do so only if he can count on being able to recover the initial expense over *five* consecutive years of cold winter weather. Upon reviewing historical data on the operation of heat pumps in various kinds of winter weather, he computes the expected annual costs of heating his home during the winter months with and without a heat pump in operation. These cost figures are shown in Table 10.29. The probabilities of experiencing a mild, normal, colder than normal, and severe winter are $0.2(1-x)$, $0.5(1-x)$, $0.3(1-x)$, and x, respectively.

a. Given that $x = 0.1$, what is the most that the homeowner is willing to pay for the heat pump?

b. If the heat pump costs \$500, how large must x be before the homeowner decides it is economically worthwhile to install the heat pump?

c. Given that $x = 0.1$, compute and interpret the expected value of perfect information (EVPI) when the heat pump costs \$500.

d. Repeat part **c** when $x = 0.15$.

56. Consider a company that manufactures computer memory chips in lots of ten chips. From past experience, the company knows that 80% of all lots contain 10% defective chips, and 20% of all lots contain 50% defective chips. If an *acceptable* (that is, 10% defective) batch of chips is sent on to the next stage of production, processing costs of

TABLE 10.29 **Expected Winter Heating Costs for Homeowner's Decision Problem**

Decision Alternatives	Mild	Normal	Colder than Normal	Severe
Purchase pump	\$420	\$590	\$720	\$900
Don't purchase pump	\$358	\$503	\$612	\$765

$10,000 are incurred. If an *unacceptable* (that is, 50% defective) batch is sent on to the next stage of production, processing costs of $40,000 are incurred. This company also has the option of reworking a batch of chips at a cost of $10,000. A reworked batch is guaranteed to be acceptable. Alternatively, at a cost of $1000, the company can test one memory chip from each batch in an attempt to determine whether the given batch is unacceptable. If a randomly selected chip is found to be defective, the batch from which the chip came is acceptable with probability $8/18$. If a randomly selected chip is found *not* to be defective, the batch from which the chip came is acceptable with probability $72/82$.

a. Determine how this company can minimize the expected total cost per batch of computer memory chips.

b. Compute and interpret the expected value of sample information (EVSI) in this decision problem.

c. Compute and interpret the expected value of perfect information (EVPI) in this decision problem.

d. Suppose now that this manufacturer's utility function of cost c per batch is $U(c) = -c^{0.5}$. Find the strategy that maximizes the manufacturer's expected utility. How does this optimal strategy compare to the optimal decision with an EMV criterion? Explain any difference in two optimal strategies.

57. Patty is trying to determine whether to take management science or statistics. If she takes management science, she believes there is a 10% chance she will receive an A, a 40% chance she will receive a B, and a 50% chance she will receive C. If Patty takes statistics, she has a 70% chance of receiving a B, a 25% chance of a C, and a 5% chance of a D. Patty is indifferent between the following two options:

- Option 1: Receiving a B for certain
- Option 2: A 70% chance at an A and a 30% chance at a D

Patty is also indifferent between the following two options:

- Option 3: Receiving a C for certain
- Option 4: A 25% chance at an A and a 75% chance at a D

In order to maximize the expected utility associated with her final grade, which course should Patty take?

58. Many men over 50 take the PSA blood test. The purpose of the PSA test is to detect prostate cancer early. Dr. Rene Labrie of Quebec conducted a study to determine whether the PSA test can actually prevent cancer deaths. In 1989, Dr. Labrie randomly divided all male registered voters between 45 and 80 in Quebec City into two groups. Two-thirds of the men were asked to be tested for prostate cancer and one-third were not asked. Eventually, 8137 men were screened for prostate cancer (PSA plus digital rectal exam) in 1989; 38,056 men were not screened. By 1997 only 5 of the screened men had died of prostate cancer while 137 of the men who were not screened had died of prostate cancer. (*Source*: *New York Times* May 19,1998)

a. Discuss why this study seems to indicate that screening for prostate cancer saves lives.

b. Despite the results of this study, many doctors are not convinced that early screening for prostate cancer saves lives. Can you see why they doubt the conclusions of the study?

59. You have just been chosen to appear on "Hoosier Millionaire"! The rules are as follows: There are four hidden cards. One says "STOP" and the other three have dollar amounts of $150,000, $200,000, and $1,000,000. You get to choose a card. If the card says "STOP," you win no money. At any time you may quit and keep the largest amount of money that has appeared on any card you have chosen, or you may continue. If you continue and choose the STOP card, however, you win no money. As an example, you might first choose the $150,000 card, then the $200,000 card, and then choose to quit and receive $200,000.

a. If your goal is to maximize your expected payoff, what strategy should you follow?

b. Suppose your utility function for an increase in cash satisfies $U(0) = 0$, $U(\$40{,}000) = 0.25$, $U(\$120{,}000) = 0.50$, $U(\$400{,}000) = 0.75$ and $U(\$1{,}000{,}000) = 1$. Are you risk averse? Explain.

c. After drawing a curve through the points in part **b**, determine a strategy that maximizes your expected utility. (Alternatively, you might want to assess and use your *actual* utility function.)

60. You are trying to determine how much money to put in your Tax Saver Benefit (TSB) plan. At the beginning of the calendar year, a TSB allows you to put money into an account. The money in the account can be used to pay for medical expenses incurred during the year. Once the TSB is exhausted, you must pay the medical expenses out of pocket. The benefit of the TSB is that money placed in the TSB is not subject to federal taxes. The catch is that any money left in the TSB at the end of the year is lost to you. Suppose the federal tax rate is 40% and your current annual salary is $50,000. You believe that it is equally likely that your medical expenses during the current year will be $3000, $4000, $5000, $6000, or $7000.

a. If you are risk neutral and want to maximize your expected disposable income, how much should you put in your TSB?

b. Suppose you assess a utility function for disposable income given by $U(x) = 0.000443x^{0.713595}$. (Who said they all have to have nice round numbers?) Are you risk averse? How much should you put in the TSB?

61. Peter is thinking of purchasing an advertising company from Amanda. At present, only Amanda (not Peter) knows the current value of the company. Peter knows, however, that there is an equal chance that the company is worth 10, 20, 30, 40, 50, 60, 70, 80, 90, or 100 million dollars. Amanda will accept an offer from Peter only if Peter bids at least the value of the company. For example, if Amanda knows the company is worth \$20 million, she will accept any bid of \$20 million or higher. As soon as Peter purchases the company, his reputation as a skilled businessman immediately increases the actual value of the company by 80%.

a. Suppose Peter is risk neutral and is considering bidding 10, 20, 30, 40, 50, 60, 70, 80, 90, or 100 million dollars. What should he bid?

b. Suppose Peter's utility function for financial gains or losses (in millions of dollars) is given by $U(x) = ((x + 82)/144)^{1.7}$. Determine whether Peter is risk averse or risk seeking and determine Peter's optimal decision.

62. Sarah Chang is the owner of a small electronics company. In 6 months a proposal is due for an electronic timing system for the 1998 Olympic Games. For several years, Chang's company has been developing a new microprocessor, a critical component in a timing system that would be superior to any product currently on the market. However, progress in research and development has been slow, and Chang is unsure about whether her staff can produce the microprocessor in time. If they succeed in developing the microprocessor (probability p_1), there is an excellent chance (probability p_2) that Chang's company will win the \$1 million Olympic contract. If they do not, there is a small chance (probability p_3) that she will still be able to win the same contract with an alternative, inferior timing system that has already been developed.

If she continues the project, Chang must invest \$200,000 in research and development. In addition, making a proposal (which she will decide whether to do after seeing whether the R&D is successful or not) requires developing a prototype timing system at an additional cost of \$50,000. Finally, if Chang wins the contract, the finished product will cost an additional \$150,000 to produce.

a. Develop a decision tree that can be used to solve Chang's problem. You can assume in this part that she is using EMV (of her net profit) as a decision criterion. Build the tree so that she can enter any values for p_1, p_2, and p_3 (in input cells) and automatically see her optimal EMV and optimal strategy from the tree.

b. If $p_2 = 0.8$ and $p_3 = 0.1$, what value of p_1 makes Chang indifferent between abandoning the project and going ahead with it?

c. How much would Chang be willing to pay the Olympic organization (now) to guarantee her the contract in the case where her company is successful in developing the contract? (This guarantee is in force only if she is successful in developing the product.) Assume $p_1 = 0.4$, $p_2 = 0.8$, and $p_3 = 0.1$.

d. Suppose now that this a "big" project for Chang. Therefore, she decides to use expected utility as her criterion, with an exponential utility function. Using some trial and error, see which risk tolerance changes her initial decision from "go ahead" to "abandon" when $p_1 = 0.4$, $p_2 = 0.8$, and $p_3 = 0.1$.

63. Suppose an investor has the opportunity to buy the following contract, a stock call option, on March 1. The contract allows him to buy 100 shares of ABC stock at the end of March, April, or May at a guaranteed price of \$50 per share. He can "exercise" this option at most once. For example, if he purchases the stock at the end of March, he can't purchase more in April or May at the guaranteed price. The current price of the stock is \$50. Each month, we assume the stock price either goes up by a dollar (with probability 0.6) or down by a dollar (with probability 0.4). If the investor buys the contract, he is hoping that the stock price will go up. The reasoning is that if he buys the contract, the price goes up to \$51, and he buys the stock (that is, he exercises his option) for \$50, he can turn around and sell the stock for \$51 and make a profit of \$1 per share. On the other hand, if the stock price goes down, he doesn't have to exercise his option; he can just throw the contract away.

a. Use a decision tree to find the investor's optimal strategy (that is, when he should exercise the option), *assuming* he purchases the contract.

b. How much should he be willing to pay for such a contract?

64. The Ventron Engineering Company has just been awarded a \$2 million development contract by the U.S. Army Aviation Systems Command to develop a blade spar for its Heavy Lift Helicopter program. The blade spar is a metal tube that runs the length of and provides strength to the helicopter blade. Due to the unusual length and size of the Heavy Lift Helicopter blade, Ventron is unable to produce a single-piece blade spar of the required dimensions, using existing extrusion equipment and material.

The engineering department has prepared two alternatives for developing the blade spar:

(1) sectioning or (2) an improved extrusion process. Ventron must decide which process to use. (Backing out of the contract at any point is not an option.) The risk report has been prepared by the engineering department. The information from it is explained below.

The sectioning option involves joining several shorter lengths of extruded metal into a blade spar of sufficient length. This work will require extensive testing and rework over a 12-month period at a total cost of $1.8 million. While this process will definitely produce an adequate blade spar, it merely represents an extension of existing technology.

To improve the extrusion process, on the other hand, it will be necessary to perform two steps: (1) improve the material used, at a cost of $300,000, and (2) modify the extrusion press, at a cost of $960,000. The first step will require 6 months of work, and if this first step is successful, the second step will require another 6 months of work. If both steps are successful, the blade spar will be available at that time, that is, a year from now. The engineers estimate that the probabilities of succeeding in steps 1 and 2 are 0.9 and 0.75, respectively. However, if either step is unsuccessful (which will be known only in 6 months for step 1 and in a year for step 2), Ventron will have no alternative but to switch to the sectioning process—and incur the sectioning cost on top of any costs already incurred.

Development of the blade spar must be completed within 18 months to avoid holding up the rest of the contract. If necessary, the sectioning work can be done on an accelerated basis in a 6-month period, but the cost of sectioning will then increase from $1.8 million to $2.4 million.

Frankly, the Director of Engineering, Dr. Smith, wants to try developing the improved extrusion process. This is not only cheaper (if successful) for the current project, but its expected side benefits for future projects could be sizable. Although these side benefits are difficult to gauge, Dr. Smith's best guess is an additional $2 million. (Of course, these side benefits are obtained only if both steps of the modified extrusion process are completed successfully.)

a. Develop a decision tree to maximize Ventron's EMV. This includes the revenue from this project, the side benefits (if applicable) from an improved extrusion process, and relevant costs. You don't need to worry about the time value of money; that is, no discounting or NPVs are required. Summarize your findings in words in the spreadsheet.

b. What value of side benefits would make Ventron indifferent between the two alternatives?

c. How much would Ventron be willing to pay, right now, for perfect information about both steps of the improved extrusion process? (This information would tell Ventron, right now, the ultimate success/failure outcomes of both steps.)

65. Ligature, Inc. is a company that does contract work for publishing companies. It specializes in writing textbooks for secondary schools. Because states such as Texas and California typically adopt only about four to eight textbooks for any given subject and grade level (from which individual schools can choose), the potential for large profits is great.

Ligature is currently negotiating a contract with Brockway and Coates (B&C), a large publishing company, to write a social studies series for grades 9–12. Actually, the development of the books is already well under way, and the only details not yet worked out concern the fee B&C will pay Ligature. Ligature has always operated on a fixed fee basis. Under this arrangement, B&C would pay Ligature its costs, in this case $4.15 million, plus 25%. Ligature would receive this payment in 6 months, at the beginning of year 1. Although this is still an option, the companies have also been discussing a royalty arrangement as an alternative.

Under the royalty plan, B&C would still pay Ligature its $4.15 million costs at the beginning of year 1, but Ligature would then receive yearly royalty payments at the ends of years 1 through 5. These payments would depend on (1) total sales over the five years, (2) the timing of sales, and (3) the negotiated royalty rate, that is, Ligature's percentage of each sales dollar. As for timing, both parties agree that 10% of total sales will be in year 1, 20% will be in each of the next 2 years, 30% will be in year 4, and 20% will be in year 5. They also estimate that the probability distribution of total sales is discrete, with possible values $25 million, $30 million, $50 million, and $70 million, and corresponding probabilities 0.10, 0.45, 0.30, and 0.15.

To guard its interests, B&C has imposed the following restriction to any royalty agreement. It places a cap on the amount Ligature can earn through the royalty scheme. Specifically, the royalties, discounted back to the beginning of year 1 at a 10% discount rate, cannot exceed 33% of Ligature's $4.15 million costs. Obviously, this limits B&C's downside exposure, regardless of the negotiated royalty rate or how well the books sell.

Ligature is interested in maximizing the NPV of its profit from this project (discounted back to the beginning of year 1), using a 10% discount rate. The following steps lead you through the required calculations to "solve" the problem. No decision tree is required for this problem.

a. The file P10_65.XLS supplies the inputs in an input section (blue border), and it has a calculation section (red border). First, calculate the upper part of the calculation section. To do so, enter any trial value for total sales in cell G8 and do the necessary calculations to eventually find (in cell G17) the NPV to Ligature from the royalty agreement. At this point, you can use any royalty rate in the RoyRate cell (C28).

b. Using the calculations from part **a**, complete the data table in the middle part of the calculation section. It should show the NPV to Ligature for any potential value of total sales. Then use these NPVs to calculate the expected NPV to Ligature in the ExpNPV cell (G27).

c. Suppose the current "offer on the table" is a 3% royalty rate. In the bottom part of the calculation section, use IF comparisons to see which arrangement, fixed fee or royalty, each party would favor.

d. Continuing part **c** (with the 3% offer on the table), what do you think the two parties will eventually agree upon? That is, will they stick with the 3% royalty rate, move to a different royalty rate, or settle on the fixed fee arrangement? Answer below cell B36.

66. The American chess master Jonathan Meller is playing the Soviet expert Yuri Gasparov in a two-game exhibition match. Each win earns a player one point, and each draw earns a half point. The player who has the most points after two games wins the match. If the players are tied after two games, they play until one wins a game; then the first player to win a game wins the match. During each game, Meller has two possible strategies: to play a daring strategy or to play a conservative strategy. His probabilities of winning, losing, and drawing when he follows each strategy are shown in Table 10.30. To maximize his probability of winning the match, what should the American do?

TABLE 10.30 Probabilities for Chess Problem

Strategy	Win	Loss	Draw
Daring	0.45	0.55	0.00
Conservative	0.00	0.10	0.90

67. Based on Balson et al. (1992). An electric utility company is trying to decide whether to replace its PCB transformer in a generating station with a new and safer transformer. To evaluate this decision, the utility needs information about the likelihood of an incident, such as a fire, the cost of such an incident, and the cost of replacing the unit. Suppose that the total cost of replacement as a present value is \$75,000. If the transformer is replaced, there is virtually no chance of a fire. However, if the current transformer is retained, the probability of a fire is assessed to be 0.0025. If a fire occurs, then the cleanup cost could be high (\$80 million) or low (\$20 million). The probability of a high cleanup cost, given that a fire occurs, is assessed at 0.2.

a. If the company uses EMV as its decision criterion, should it replace the transformer?

b. Perform a sensitivity analysis on the key parameters of the problem that are difficult to assess, namely, the probability of a fire, the probability of a high cleanup cost, and the high and low cleanup costs. Does the optimal decision from part **a** remain optimal for a "wide" range of these parameters?

c. Do you believe EMV is the correct criterion to use in this type of problem involving environmental accidents?

68. Based on Mellichamp et al. (1993). Construction equipment managers typically have many large pools of engines, transmissions, and other equipment units to maintain. One approach to this maintenance is to use oil analysis, where the oil from any of these is subjected periodically to an inspection. These inspections can sometimes signal an impending failure (for example, too much iron in the oil), and preventive maintenance is then performed (at a relatively low cost), eliminating the risk of failure (failure would result in a relatively high cost). However, oil analysis costs money, and it is not perfect. That is, it can indicate that a unit is defective when in fact it is not about to fail, and it can indicate that a unit is nondefective when in fact it is about to fail. As a possible substitute for oil analysis, the company could simply change the oil periodically, thereby reducing the probability of a failure.

Suppose the company has four alternatives: (1) do nothing, (2) use oil analysis only, (3) replace oil only, or (4) replace oil and do oil analysis. For option (1) the probability of a failure is p_1, and the cost of a failure is C_1. For option (2), the probability of a failure remains at p_1. If the unit is about to fail, the oil analysis will indicate this with probability $1-\alpha$; if the unit is not about to fail, the oil analysis will indicate this with probability $1-\beta$. (Therefore, α and β are the error probabilities of the oil analysis.) The oil analysis itself costs C_2, and if it indicates that a failure is about to occur, the oil will be changed, at cost C_3, and preventive maintenance will be performed. The cost of maintenance to restore a unit that is about to fail is C_4, whereas the cost of maintenance for a unit that is not about to fail is C_5. The only difference between options (3) and (4) is that the probability of a failure decreases to p_2 after changing the oil. The values of these parameters for a particular class of units (engines in light trucks, say) appear in Table 10.31 (page 556).

TABLE 10.31 Parameters for Oil Analysis Problem

Parameter	Value
p_1	0.10
p_2	0.04
α	0.30
β	0.20
C_1	\$1200.00
C_2	\$20.00
C_3	\$14.80
C_4	\$500.00
C_5	\$250.00

a. For these parameters, develop a decision tree to find the company's best decision and the corresponding expected cost.

b. If the company has 500 units, what should it do? What is the expected cost for the entire fleet?

c. Suppose that the company has different types of units. For example, the cost of an oil change might be higher for some, or the cost of a failure might be higher or lower. Run a sensitivity analysis on any of the parameters you believe might be "key" parameters and see whether the optimal decision changes in ways you would anticipate.

69. Based on Hess (1993). A company that is heavily involved in R&D projects believes it might have the potential to develop a very lucrative commercial product that would (if successful) reduce pulp mill water pollution. At the current stage, however, everything is quite uncertain, and the company is trying to decide whether to go ahead with its R&D or abandon the product. The following are the primary risks:

- Would market tests confirm that there is a significant market for the product?
- Could the company develop a new process for making this product—that is, is it technically feasible?
- Even if there is a significant market and the process is technically feasible, would the company's board sanction the new plant capital necessary to produce the product on a commercial scale?
- Assuming the answers to the above questions are all yes and the plant is built, would the venture turn out to be successful?

We assume that each of these questions has a yes or no answer. The probabilities of yes answers are shown in Table 10.32. The plus-or-minus value indicates the company's uncertainty about the true probabilities.

TABLE 10.32 Probabilities for Water Pollution Problem

Event	Probability
Significant market	0.6 ± 0.15
Technically feasible	0.6 ± 0.15
Board sanctions plant expenditures	0.8 ± 0.2
Commercial success	0.8 ± 0.2

The primary economic factors are the following:

- the research expenses to identify a new production process for the product
- the marketing development cost to determine whether there is a significant market
- the process development costs, including presanction engineering
- the commercial development costs, both before and after the board's sanction
- the venture value (net present value) if successful

The estimates of these values are shown in Table 10.33. Again, the plus-or-minus values indicate the company's considerable uncertainty about the values. All values are in millions of dollars.

The timing of events is as follows:

- Decide whether to abandon product now. (This is really the only nontrivial decision the company will make.) If not, then:
- Spend on research and marketing development. If marketing development indicates an insignificant market for the product *or* research indicates that the process is technically infeasible, cut expenses and quit. Otherwise:
- Spend on process and commercial development. If company board then declines to sanction money for plant, cut expenses and quit. Otherwise:
- Spend on further commercial development. By this time, the company has made all of its decisions. If the venture turns out to be a commercial success, then it gains the venture value for a success (less expenses so far). Otherwise, the company has lost the money spent so far, but that is all.

Analyze the company's problem. Obviously, with the high degree of uncertainty, sensitivity analysis is the key. Note that there are many uncertainties about the input parameters in Tables 10.32 and 10.33. In fact, there are far too many to allow you to try every combination. Therefore, just try a few combinations that you believe might be the most important.

TABLE 10.33 Monetary Estimates for Water Pollution Problem

Expense or Gain	Net Present Value
Research expense	\$0.8 ± 25%
Market development expense	\$0.2 ± 25%
Process development expense (presanction)	\$3.0 ± 25%
Commercial development expense (presanction)	\$0.5 ± 25%
Commercial development expense (postsanction)	\$1.0 ± 25%
Value if successful	\$25.0 ± 50%

CASE 10.1

GMC Motor Company II

This case is a continuation of GMC I (from Chapter 6). Management at GMC is generally pleased with the modeling effort that has been done for capacity planning in the coming year. However, some managers have asked about the effect demand forecasts for the second year out could have on the recommended *strategy*.

Although demand forecasts for the coming year are considered to be quite reliable, forecasts two or more years in the future have been less accurate. Accordingly, analysts at GMC formulate several demand scenarios in the future, and assign probabilities to each scenario. The situation for the coming two years is summarized in Table 10.34.

Three demand scenarios are possible in the second year. Scenario A corresponds to a robust economic expansion and increasing market share for GMC cars. Scenario B represents little change from the first year, although there is a relative shift away from the smaller Lyra to the larger Libra and Hydra models. Scenario C represents an economic recession and decreased demand for all car lines. In scenario C, the decrease in demand for Libras and Hydras is larger than for the economical Lyras. Analysts give scenario A a slightly higher probability of occurring than scenarios B and C.

Management at GMC wants to consider all possible configurations of capacity in the next two years. As before, the Lyra and/or Libra plants can be retooled, *but retooling can be done in either the first or second year*. Because of the enormous costs of changing a plant configuration, a plant that is retooled in the first year cannot be returned to its original configuration in the second year. The costs and characteristics of the original and retooled plants are the same in either year. For convenience, these are repeated in Table 10.35.

In addition to selecting the plant configurations for each year, GMC needs to determine the production plan at each plant in each year. The sequence of events and decisions is as follows. At the beginning of a year, GMC must decide on the plant configurations. Demand occurs during the year, and based on the observed demand, GMC plans its production accordingly. For example, in the second year, GMC must decide on its plant configurations *before* the demand scenario is revealed, but can determine its production plan *after* the demand scenario is revealed. This sequence of events is consistent with the relative time periods involved. Reconfiguring a plant is a major undertaking that must be planned in advance, so this decision must be made before the demand scenario is revealed. Production during a year can be altered to best meet the demand as it develops during the year. For modeling purposes, the production decision can be made after the demand scenario is revealed. Also, no inventory is carried from one year to the next.

The demand diversion matrix is assumed to be constant for both years. For convenience, it is repeated in Table 10.36.

Questions

GMC wants to decide whether to retool the Lyra and Libra plants in each of the coming two years. In addition, GMC wants to determine its production plan at each plant for each year. Based on the previous data, formulate a mixed integer programming model for solving GMC's production planning–capacity expansion problem for the coming two years. Assume that GMC's objective is to maximize total average profit for the two years. For simplicity, assume that no discounting of profits is done for the second year.

In the past, GMC had solved problems separately for each scenario. The three optimal solutions were compared and then a final decision was made. What are the three optimal solutions corresponding to each scenario? (For example, assuming that scenario A occurs with probability 1.0, what is the optimal solution? Then repeat for scenarios B and C.) How do the three separate optimal solutions compare to the overall optimal solution found before?[8]

[8]*Acknowledgment*: The idea for GMC I and II came from Eppen et al. (1989).

TABLE 10.34 Demand Forecasts and Probabilities for GMC Case Study

Model	First Year	Second Year Scenario A	Second Year Scenario B	Second Year Scenario C
Lyra	1400	1700	1300	1300
Libra	1100	1500	1200	800
Hydra	800	1100	850	600
Probability	1	0.4	0.3	0.3

TABLE 10.35 Plant Characteristics for GMC Case Study

	Lyra	Libra	Hydra	New Lyra	New Libra
Capacity (in 1000s)	1000	800	900	1600	1800
Fixed cost (in $millions)	2000	2000	2600	3400	3700
Profit Margin by Car Line (in 1000s)					
Lyra	2	–	–	2.5	2.3
Libra	–	3	–	3	3.5
Hydra	–	–	5	–	4.8

TABLE 10.36 Demand Diversion Matrix for GMC Case Study

	Lyra	Libra	Hydra
Lyra	–	0.3	0.05
Libra	0	–	0.10
Hydra	0	0.0	–

CASE 10.2

Jogger Shoe Company

The Jogger Shoe Company is trying to decide whether to make a change in its most popular brand of running shoes. The new style would cost the same to produce, and it would be priced the same, but it would incorporate a new kind of lacing system that (according to its marketing research people) would make it more popular. There is a fixed cost of $300,000 of changing over to the new style. The unit contribution to before-tax profit for either style is $8. The tax rate is 35%. Also, because the fixed cost can be depreciated and will therefore affect the after-tax cash flow, we need a depreciation method. We assume it is straight-line depreciation.

The current demand for these shoes is 190,000 pairs annually. The company assumes this demand will continue for the next 3 years if the current style is retained. However, there is uncertainty about demand for the new style, if it is introduced. The company models this uncertainty by assuming a normal distribution in year 1, with mean 220,000 and standard deviation 20,000. The company also assumes that this demand, whatever it is, will remain constant for the next 3 years. However, if demand in year 1 for the new style is sufficiently low, the company can always switch back to the current style and realize an annual demand of 190,000. The company wants a strategy that will maximize the expected net present value (NPV) of total cash flow for the next 3 years, where a 15% interest rate is used for the purpose of calculating NPV.

CASE 10.3

Westhouser Paper Company

The Westhouser Paper Company in the state of Washington currently has an option to purchase a piece of land with good timber forest on it. It is now May 1, and the current price of the land is $2.2 million. Westhouser does not actually need the timber from this land until the beginning of July, but its top executives fear that another company might buy the land between now and the beginning of July. They assess that there is 1 chance out of 20 that a competitor will buy the land during May. If this does not occur, they assess that there is 1 chance out of 10 that the competitor will buy the land during June. If Westhouser does not take advantage of its current option, it can attempt to buy the land at the beginning of June or the beginning of July, provided that it is still available.

Westhouser's incentive for delaying the purchase is that its financial experts believe there is a good chance that the price of the land will fall significantly in one or both of the next two months. They assess the possible price decreases and their probabilities in Tables 10.37 and 10.38. Table 10.37 shows the probabilities of the possible price decreases during May. Table 10.38 shows the *conditional* probabilities of the possible price decreases in June, *given* the price decrease in May. For example, if the price decrease in May is $60,000, then the possible price decreases in June are $0, $30,000, and $60,000 with respective probabilities 0.6, 0.2, and 0.2.

If Westhouser purchases the land, it believes that it can gross $3 million. (This does not count the cost of purchasing the land.) But if it does not purchase the land, it believes that it can make $650,000 from alternative investments. What should the company do?

TABLE 10.37 Distribution of Price Decrease in May

Price Decrease	Probability
$0	0.5
$60,000	0.3
$120,000	0.2

TABLE 10.38 Distribution of Price Decrease in June

Price Decrease in May					
$0		$60,000		$120,000	
June Decrease	Probability	June Decrease	Probability	June Decrease	Probability
$0	0.3	$0	0.6	$0	0.7
$60,000	0.6	$30,000	0.2	$20,000	0.2
$120,000	0.1	$60,000	0.2	$40,000	0.1