What is Time Phasing?

Time phasing a cost estimate refers to spreading, or allocating, the costs implied by the estimate into the years (typically fiscal years) in which those costs are expected to occur. The program schedule provides the data as to how the costs might be spread. Time phasing may also be based upon historical data (e.g., spend profiles of other comparable programs), or even theoretical data (e.g., based upon well-defined properties of probability distributions known to approximate spend profiles).

The scope of this CLM is limited to time phasing in the context of cost estimating, and thus, of expenditure profiles.

Popup Text

Time Phasing

Time Phasing is the distribution of costs over a period of years, based on a prediction of when the costs are expected to be incurred.

Time Phasing and the Cost Estimating Process

The depiction shown illustrates how time phasing fits into the overall cost estimating process.

Calculate Life Cycle Cost Estimate (in base year dollars)

↓

Time-phase the estimate by spreading costs into the years in which the costs are expected to occur

↓

Appropriate the estimate by assigning an appropriation to each cost element-year combination

↓

Escalate the estimate using appropriation-specific inflation indices to create a then-year dollar/budgetary estimate

D

Long Description

1. Calculate Life Cycle Cost Estimate (in base year dollars)
2. Time-phase the estimate by spreading costs into the years in which they are expected to occur
3. Appropriate the estimate by assigning an appropriation to each cost element-year combination
4. Escalate the estimate using appropriation-specific inflation indices to create a then-year dollar/budgetary estimate

Time Phasing and the Cost Estimating Process, Cont.

Time phasing occurs after traditional cost estimating is completed. It takes the base year denominated life cycle cost estimate (LCCE) as an input. Time phasing is applied to base year dollars, but affects the then-year dollar cost estimate.

In the next two steps, the time phase estimate is appropriated. In other words, an account, such as "Aircraft Procurement, Navy (APN)", is designated for every nonzero cost element in every fiscal year.

Finally, appropriation-specific inflation indices (in this example, APN indices) are used to escalate, or inflate, the estimate. This creates a then-year dollar denominated estimate that serves as a starting point for the budgeting process.

Popup Text

Life Cycle Cost Estimate (LCCE)

A Life Cycle Cost Estimate (LCCE) is the estimated cost of developing, producing, deploying, maintaining, operating and disposing of a system over its entire lifespan. The estimate is prepared for major program reviews to include Milestone A, B, and C. It's used to acquire funding for a system throughout its lifespan.

Why is Time Phasing Important?

How an estimate is time phased affects the actual dollar value of the estimate itself (when that estimate is expressed in then-year/inflated/escalated dollars).

Time phasing also:

- Supports budgeting because costs must be mapped to appropriations and years
- Becomes the basis of the resource-loaded schedule
- Creates an initial "spend plan" for the executing organization

Time Phasing Example

Consider an example in which there is a 3% annual inflation and $100 is spent in the first year. The escalated cost estimate is also $100. But what if that $100 were to be spread evenly over three years?

The escalated estimate is:

$$(33.33)+(33.33*1.03)+(33.33*1.03^2) = 33.33 + 34.33 + 35.36 = \$103.02$$

$103.02 is 3.02% higher than the original estimate of $100. Pushing the cost "further to the right" increases the escalated cost estimate, while moving costs "to the left" (i.e., earlier) decreases the estimate. Of course, this presumes positive inflation year over year.

Commonly Used Time Phasing Methods

Several commonly used time phasing methods include:

1. Level-Loaded Method
2. Throughput Methods
   - Base Year
   - Then-Year
   - Same Year
3. Schedule-Based Methods
   - Incremental Quantities
   - Cumulative Quantities
4. Trapezoid Method
5. Learning Curve Methods
   - Cumulative Average Unit Cost (CAUC)
   - Unit Theory (UT)
   - Unit Theory with Rate Adjustment (UTRA)
6. Probability Distribution-Based Methods
   - Beta and PERT Beta
   - Rayleigh and Weibull

The methods are listed roughly in order of increasing complexity. Some methods are logically grouped together because they are conceptually similar. Note that some methods are special cases, or generalizations, of other methods. This means that two or more of the above methods are equivalent under certain assumptions.

Method 1: Level-Loaded Method Overview

The Level-Loaded Method is simple to apply and understand. The basic concept involves the analyst specifying a start year, end year, and amount (or factor) to apply to each year between, and inclusive of, the start and end year. The constant cost is applied to each year. This method is most often used when amounts do not change by year.

For example, if fuel costs and fuel consumption are each known for an upcoming five year period, then one might reasonably 'level-load' fuel costs across those five years. Say the Program Office has "locked in" a cost of $3.00 per gallon for jet fuel for the next five years. It is also known that each aircraft will consume 10,000 gallons of fuel per year during that five year period. The time phased costs for fuel each year would be:

$3.00 * 10,000 = $30,000 (each year per aircraft)

Popup Text

Level-Loaded Method

Level-Loaded Method is the fundamental time phasing approach in which the analyst specifies a start year, end year, and amount to place in each year between (and inclusive of) the start and end years. Then, that constant cost is simply applied to each year. This approach is used where the parameters for a given unit of measure are well-defined and stable.

Method 1: Level-Loaded Method Strengths and Limitations

Strengths (+)	Limitations (-)
• Easy, straightforward, and very simple to understand, use, and explain • Requires virtually no mathematical calculations • Easy to defend when costs, quantities, or factors are known (or can be reasonably assumed) to be constant over time	• Cannot be used when costs, quantities, or factors vary over time

How Time Phasing Affects Budgetary Cost - Simulation Exercise

D

Long Description

How Time Phasing Affects Budgetary Costs

If you have $100M and you spend the $100M in the first year, you are not concerned with inflation in years 2 and 3.

If you are buying a product for $100M and you want to spread the cost evenly over 3 years...

Then the $100M is evenly spread over 3 years at $33.33M per year.

With the impact of inflation, budget $33.33M in base year dollars with a 3% inflation rate for years 2 and 3. For year 2, $34.44M is needed.

For year 3, you need to account for the compounded inflation for Years 1 and 2. Assuming the 3% inflation, multiply 33.33 by 1.609, resulting in an estimate $35.36M for Year 3. When all three years are added together, the total is $103.03M.

If you have a three year budget but don't spend any of the $100M until Year 3, you will face an inflation premium.

Knowledge Review

Which of the following cases might the analyst **NOT** reasonably use the Level-Loaded Method to estimate the cost of?

- [ ] A constant number of crew, working a constant number of hours, on a constant number of ships, for a fixed period of years
- [x] Aircraft procurement costs, based on a deployment schedule with different quantities in each year
- [ ] A constant number of program office staff, working a constant number of hours per year, for a fixed period of year
- [ ] Acquisition costs of 100 lbs per year of raw steel, with the assumption of constant cost per pound over a certain time frame

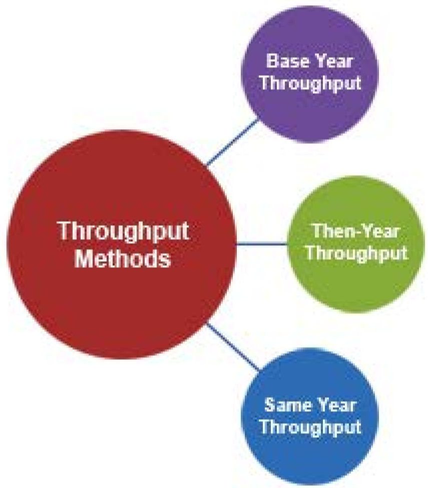
Check Answer

The factors of **aircraft procurement costs, based on a deployment schedule with different quantities each year** are not constant, making using the Level-Loaded Method a poor choice for time phasing estimates. For Answer Options A and C, the then-year dollar cost will not be constant over time because inflation affects wages of crew and program office staff. However, because time phasing applies to the base-year denominated cost estimate, the Level-Loaded Method is still appropriate.

Method 2: Throughput Methods Overview

The basic concept of the Throughput Method is that the analyst already knows the amounts (costs) to place in each year and places them accordingly. The analyst must specify amounts by year and whether the amounts are in base year, same year, or then-year dollars because this specification affects the interpretation of results.

The throughput family of methods is most suitable when costs are known (actuals), or costs can be extrapolated from known, actual costs. When costs are determined by "hard" constraints, as is often the case with "level of effort" tasks, Throughput Methods may be suitable as well.



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Popup Text

Throughput Method

The Throughput Time Phasing Method is used when the analyst already knows not just the cost in question, but how that cost should be spread over time. In such a case, the analyst still needs to specify whether the amounts are in base year, same year, or then-year dollars, because this specification affects the interpretation of results. The "throughput" family of methods is most suitable when costs are known, or are being extrapolated from known costs.

Long Description

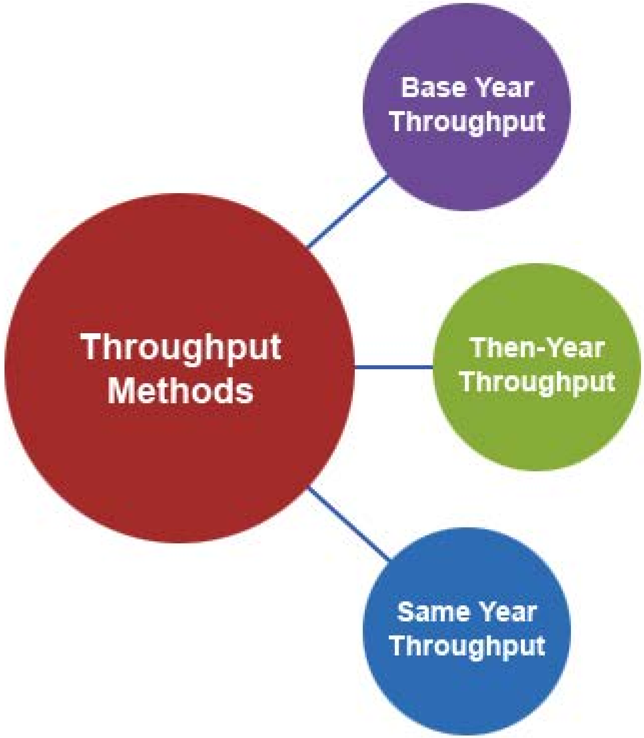
A large circle labeled Throughput Methods with three small circles connected to the larger circle each labeled – Base Year Throughput, Then-Year Throughput, and Same Year Throughput.

Method 2: Throughput Methods Examples

The three Throughput Methods are displayed here.

The purpose of distinguishing between these three types of Throughput Methods is to ensure that the analyst is mindful of the inflationary assumptions implicit in any throughput phasing method. The analyst must specify not only the amount per year, but the types of dollars implicit in those amounts.

Select each of the Throughput Methods in the diagram for more information.



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Popup Text

Base Year Throughput

Under the Base Year Throughput Method, the analyst knows the base year dollar denominated cost in each year, but the base year does not change. For example, the amounts in all years might be expressed in FY2010 dollars. Say the FY2010 base year dollar wage of a representative staff member of a Program Office is known, but the wage in FY2011 dollars (and all other years) is not known. The analyst might assume that the wage will grow by 2% so that the base year of FY2010 cost estimate grows at the same amount. The Program Office staff member earns a fully loaded wage of $100/hour in FY2010 dollars in every year, plus an average 2% merit-based increase above inflation. The base year FY2010 dollar cost estimate would be $100, $102, $104.4....etc.

Then-Year Throughput

An example of the Then-Year Throughput Method occurs when the analyst knows (or is estimating via extrapolation) actual costs. Actual costs as reported on typical Contractor Performance Reports (CPRs) are in then-year dollars, so they must be treated as such. Time phasing and escalation are done in the same step, rather than different steps. For example, a program has experienced actual costs that start at $100 and increase by 5% per year in then-year dollars and the analyst wishes to extrapolate this trend into the future. The then-year dollar cost estimate would be $100, $105, $110.25.... etc.

Contractor Performance Reports (CPRs)

The Contractor Performance Report (CPR) provides contract cost and schedule performance data that is used to identify problems early on an acquisition contract and forecast future contract performance in Earned Value Management. It's meant to convey information about the performance of a program or contract. The CPR is also the primary means of documenting the ongoing communication between the contractor and the program manager to report cost and schedule trends.

Same Year Throughput

Same Year Throughput is highly unusual and would be used very rarely. However, it is a supported method in the Automated Cost Estimating Integrated Tools (ACEIT) software, and is worth mentioning for completeness. It is equivalent to the Base Year Throughput Method in that the amounts in every year are in base year dollars. However a different base year is assumed for each year, specifically, the year in which the cost occurs is always the base year. So FY2010 amounts are in base year FY2010 dollars, FY2011 amounts are in base year FY2011 dollars, etc. Multiple Cost Estimating Relationships (CERs) apply to the same cost element in different years. This is different than then-year dollars because the amounts are not budgetary and use raw rather than weighted inflation indices. That is, they do not account for outlay profiles, or the fact that procurement must be fully funded in the year of obligation. The analyst places each CER output in its appropriate year. Inflation conversions are necessary to convert these amounts to either (constant) base year or then-year dollars.

Automated Cost Estimating Integrated Tools (ACEIT)

Automated Cost Estimating Integrated Tools (ACEIT) is a family of applications that support program managers and cost/financial analysts during all phases of a program's life-cycle. ACEIT applications are the premier tool for analyzing, developing, sharing, and reporting cost estimates, providing a framework to automate key analysis tasks and simplify/standardize the estimating process.

Cost Estimating Relationship (CER)

Cost Estimating Relationship (CER) is a mathematical relationship that defines cost as a function of one or more parameters such as performance, operating characteristics, physical characteristics, etc.

Long Description

A large circle labeled Throughput Methods with three small circles connected to the larger circle each labeled – Base Year Throughput, Then-Year Throughput, and Same Year Throughput.

Method 2: Throughput Methods Strengths and Limitations

Strengths (+)	Limitations (-)
• Simple to understand and explain: the premise behind these methods is that time phased costs are already known, so no assumptions need to be made • Does not require sophisticated mathematical calculations • Non-controversial: if time phased costs are known, then there is no uncertainty as to how to spread costs by year	• Cannot be used when time phased costs are not known • Costs by year can be inaccurate, even when a phasing algorithm is known and properly used (because there may be errors in the underlying cost estimate being phased) • Precise inflation treatments of the throughput amounts can be complex

Normalization for inflation, while beyond the scope of this CLM, is an important topic and cannot be ignored in this context. When the Then-Year Throughput Method is used, costs have already been escalated and the estimator should be mindful not to escalate twice. The Same Year Throughput Method is more complex because those amounts must be either deflated (using raw inflation indices) or inflated (using weighted indices) to be converted to base year or then-year amounts.

Method 2: Throughput Methods Simulation Exercise

The tables on the right show the interpretation of throughput amounts in base year, then year, and same year dollars.

The same throughput amounts and inflation are used in all cases, but the interpretation of throughput amounts as base year, then-year, or same year varies. As a result, the FY2010 cost ranges from $293.55M to $306.04M, and the then-year cost ranges from $306.04M to $319.25M.

	2010	2011	2012	Total
Throughput Amount	$100.00	$102.00	$104.04	
Interpret as	**Base Year**			
Raw Inflation Indices	1.0000	1.0300	1.0609	
Weighted Inflation Indices	1.0121	1.0425	1.0737	
Cost, FY2010 $M	$100.00	$102.00	$104.04	**$306.04**
Cost, Then-Year $M	$101.21	$106.33	$111.71	**$319.25**

	2010	2011	2012	Total
Throughput Amount	$100.00	$102.00	$104.04	
Interpret as	**Then-Year**			
Raw Inflation Indices	1.0000	1.0300	1.0609	
Weighted Inflation Indices	1.0121	1.0425	1.0737	
Cost, FY2010 $M	$98.81	$97.85	$96.90	**$293.55**
Cost, Then-Year $M	$100.00	$102.00	$104.04	**$306.04**

	2010	2011	2012	Total
Throughput Amount	$100.00	$102.00	$104.04	
Interpret as	**Same Year**			
Raw Inflation Indices	1.0000	1.0300	1.0609	
Weighted Inflation Indices	1.0121	1.0425	1.0737	
Cost, FY2010 $M	$100.00	$99.03	$98.07	**$297.10**
Cost, Then-Year $M	$101.21	$103.23	$105.30	**$309.74**

Knowledge Review

Suppose that you have an actual contractor spend plan, i.e., a prediction of what invoices will show in each year, and you wish to use it as the basis for a time phased cost estimate. Which method would be most appropriate?

- [ ] Level-Loaded
- [ ] Base Year Throughput
- [ ] Then-Year Throughput
- [x] Same Year Throughput

Check Answer

Actual invoices are in same year dollars, as they reflect expenditures to occur in each year. The spend plan is also in same year dollars, so **Same Year Throughput** would be the most appropriate method.

Knowledge Review

Which Throughput Method is most conceptually consistent with the role of time phasing in the overall cost estimating process?

- [x] Base Year Throughput
- [ ] Then-Year Throughput
- [ ] Same Year Throughput

Check Answer

Time phasing properly occurs when the estimate is in base year dollars (where every year's dollars are expressed in the same base year). This means that the **Base Year Throughput Method** is the only throughput method that is conceptually consistent with the GAO Guide. The other two methods assume that the analyst has estimated costs that are in some other type of dollars.

Knowledge Review

Which Throughput Method is equivalent to the Level-Loaded Method, in the special case that the amounts in all years are the same?

- [x] Base Year Throughput
- [ ] Then-Year Throughput
- [ ] Same Year Throughput

Check Answer

The Level-Loaded Method assumes base year dominated inputs, so if the inputs are constant across years, then **Base Year Throughput** and Level-Loaded Method are equivalent.

Method 3: Schedule-Based Methods Overview

The Schedule-Based Methods use the Cost Analysis Requirements Description (CARD), or other technical baseline documents, to ascertain the actual program schedule, and use that schedule as the basis for time phasing total costs. Analysts should independently analyze the development or implementation schedule for reasonableness based upon known schedule estimating relationships (SERs) or historical data. Since independent verification of the schedule is beyond the scope of this CLM, we assume the schedule has been independently vetted already for the purpose of this time phasing discussion.

The quantities per year should be specified when using Schedule-Based Methods (e.g., deployment or implementation schedule). The key distinction between Schedule-Based Methods is the type of quantities with which one is concerned: incremental or cumulative. For acquisition costs, incremental quantities are of interest because assets are only acquired once. For operations and maintenance costs, cumulative quantities are of interest because the assets must be operated and maintained in each year that they are active. Cumulative quantities should also be reduced according to a well-defined retirement schedule.

Schedule-Based Methods are often used for the estimation of hardware acquisition costs and the estimation of hardware operations and maintenance costs.

Popup Text

Schedule-Based Methods

The Schedule-Based Time Phasing Method is used when the analyst uses the CARD, or another official technical baseline that contains the actual program schedule, as the basis for time phasing total costs.

Cost Analysis Requirements Description (CARD)

Cost Analysis Requirements Description (CARD) is a description of the salient features of the acquisition program and of the system itself. It is the common description of the technical and programmatic features of the program that is used in preparing the Program Office Estimate (POE), Component Cost Estimate (CCE), and independent Life Cycle Cost Estimates (LCCEs).

Method 3: Schedule-Based Methods Example

Consider a case in which the analyst wishes to estimate airframe costs. The analyst knows 10 aircraft are being procured: 2 in the first year, 3 in the second year, and 5 in the third year. The total acquisition cost is $10M.

The incremental quantities (2, 3, and 5) become the basis of the acquisition cost time phasing, while the cumulative quantities (2, 5, and 10) would be the basis for operations and maintenance time phasing.

Popup Text

Incremental Quantities

- The analyst is estimating acquisition costs. There are neither learning curves nor rate effects (constant unit cost). The percentage of total cost assigned to each year (in base year dollars) equals the percentage of aircraft deployed in that year.
- The time phased costs are (2M, $3M, $5M).

Cumulative Quantities

- The analyst is estimating operations and maintenance costs. It costs $100K per year to maintain one aircraft. Then the cumulative, not incremental, quantities are relevant.
- The cumulative quantities are 2, 5, and 10.
- The time phased costs are $200K, $500K, and $1M.
- These costs must be ramped down as the aircraft are retired.

Method 3: Schedule-Based Methods Strengths and Limitations

Strengths (+)	Limitations (-)
• Intrinsically linked to the program's technical/schedule baseline • Easy to document and defend • Straightforward mathematical calculations • Can account for leads and lags in program schedule in order to model a situation in which costs are incurred before (and after) assets are procured	• Link to schedule means that time phased costs are only as realistic as the schedule on which they are based; the analyst inherits all inaccuracies and uncertainty • Does not explicitly account for learning or rate effects (assumes constants unit cost unless learning curves are incorporated) • Updates are not automatic: cost and schedule must be linked so that cost analyst updates the estimate every time the schedule baseline changes

Method 3: Schedule-Based Methods Simulation Exercise

Many factors can affect the Schedule-Based Methods. Time phased costs are impacted by changes in acquisition quantities (the CARD), retirement quantities, and any lead/lag amount (in years).
Lead and lag changes are indicated in positive and negative numbers; lead is positive while lag is negative. A lead entry (positive value) means that costs are experienced before the items are procured while a lag entry (negative value) means that the costs are experienced after the items are procured. A default value of '0' indicates no lead or lag.
In the example below, there is no lead time required to budget for the purchase of an aircraft. If advance procurement were used, you would need to request the funds in 2009 for the appropriate portion of a 2010 aircraft.

Lead or Lag (years):	0														
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	TOTAL
CARD Quantities:	2	3	5	0	0	0	0	0	0	0	0	0	0	0	10
Cumulative Quantities:	2	5	10	10	10	10	10	10	10	10	10	10	10	10	127
Number Retired:	0	0	0	0	0	0	0	0	0	2	2	2	2	2	10
Cumulative Number Retired:	0	0	0	0	0	0	0	0	0	2	4	6	8	10	30
Active Quantities:	2	5	10	10	10	10	10	10	10	8	6	4	2	0	97
Total acquistion cost (FY 2010$M):	$10.0														
Unit O&M cost (FY 2010$M:	$0.1														
Time Phased Acq Cost (FY 2010$M):	$2.0	$3.0	$5.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	$0.0	**$10.0**
Time Phased O&M Cost (FY 2010$M):	$0.2	$0.5	$1.0	$1.0	$1.0	$1.0	$1.0	$1.0	$1.0	$0.8	$0.6	$0.4	$0.2	$0.0	**$9.7**
Time Phased LCC (FY 2010 $M):	$2.2	$3.5	$6.0	$1.0	$1.0	$1.0	$1.0	$1.0	$1.0	$0.8	$0.6	$0.4	$0.2	$0.0	**$19.7**

D

Long Description

Screenshot of an excel sheet that is used to calculate time phased costs. The numbers of the CARD Quantities and Numbered Retired rows are highlighted for the years 2010 through 2023; all numbers are positive and range from 0 to 2.

Knowledge Review

When cumulative quantities are downward-adjusted to incorporate a retirement schedule, what is the effect on cost and in what phase does this occur?

- [ ] Increases cost, Acquisition Phase
- [ ] Increases cost, O&M Phase
- [ ] Decreases cost, Acquisition Phase
- [x] Decreases cost, O&M Phase

Check Answer

By taking assets out of the field, asset retirements **decrease the total operations and maintenance requirements**. Since everything is acquired only once, retirement schedules do not affect acquisition costs.

Knowledge Review

If quantities are constant in each year, the Schedule-Based Methods (using incremental quantities) are equivalent to which other method?

- [ ] Level-Loaded, for all costs
- [ ] Then-Year Throughput, for all costs
- [x] Level-Loaded, for acquisition costs only
- [ ] Then-Year Throughput, for acquisition costs only

Check Answer

The question assumes that quantities are constant in every year. If unit costs are also constant (as in the Schedule-Based Methods), it is equivalent to the **Level-Loaded Method**. However, **this is only true for acquisition costs** because operation and maintenance costs are determined by active quantity, which would then not be constant by year.

Method 4: Trapezoid Method Overview

The Trapezoid Method enables the analyst to time phase costs for a situation involving 'ramp up' (increasing cost), 'steady state' (constant cost), and 'ramp down' (decreasing cost) period. To do this, the analyst specifies:

- The length (in years) and rate (in dollars or quantities per year) of the 'ramp up' time
- The length of 'steady state' (by definition, its rate is zero)
- The length of 'ramp down' time (the rate of ramp down time is implied by the total cost, which by assumption is already calculated

This method is most often used to model a situation with 'ramp up', 'steady state', and 'ramp down' periods. An example would be a staffing model in which the contractor (or program office) needs time to hire staff incrementally, then executes at 'peak staffing' and ultimately ramps back down to zero staff.

Popup Text

Trapezoid Method

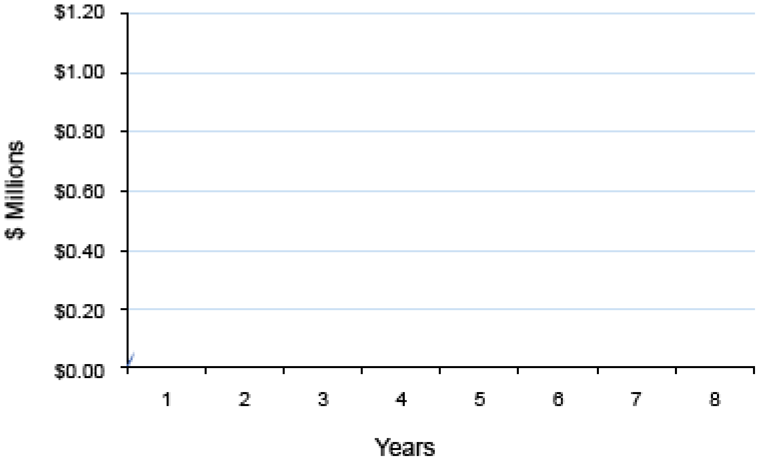
The Trapezoid Time Phasing Method is used when the analyst models a situation in which there are "ramp up," "steady state," and "ramp down" periods with respect to cost. To do this, the analyst must specify the length (in years) and rate (in dollars or quantities per year) of "ramp up" time, the length of "steady state" time (by definition, its rate is zero), and the length of "ramp down" time (the rate of ramp down time is implied by the total cost, which by assumption is already calculated).

Method 4: Trapezoid Method Example

While estimating program office staffing costs, the analyst knows the life cycle cost associated with a particular staffing cost element, and how that cost was calculated. The total staffing cost is $4M (FY2010 dollars) and is estimated as $100K per person-year multiplied by 40 person-years. Based on the staffing requirement laid out in the CARD, the analyst models the phasing of the cost using the Trapezoid Method.

Using historical experience with similar programs, the analyst specifies the following:

- 2 years of hiring 5 people per year as the 'ramp up'
- 1 year of peak staff (10 people) as 'steady state'
- 4 years of attrition/retirement as 'ramp down'



The slope in each phase is linear, meaning the associated trapezoid is four connected line segments, two of which are horizontal and parallel to each other.

Using the Trapezoid Method, the time phased costs are found to be {$0.25M, $0.75M, $1.0M, $0.875M, $0.625M, $0.375M, $0.125M}.

D

Long Description

A line graph with the x axis labeled in years 1 through 8 and a y axis in millions of dollars in increments of twenty from zero dollars to 1.2M. The trapezoid method shows the following a 2 years of hiring 5 people/year as "ramp up" and 2 years of peak staff (10 people) as "steady state" then a 4 years of attrition/retirement as "ramp down".

Method 4: Trapezoid Method Strengths and Limitations

Strengths (+)	Limitations (-)
• Allows the analyst significant flexibility; no obligation to use the same values in each year (as in the Level-Loaded Method), no need to know the phased results ahead of time (as in the Throughput Methods), no locking into the program schedule (as in the Schedule-Based Methods) • Allows choice of rates and lengths associated with 'ramp up', 'steady state', and 'ramp down' phases • Intuitive method with respect to staffing costs; contractors and program offices alike are often in the position of needing to hire staff incrementally, then executing work at a certain peak staff level, and ramping down the staff as the project winds down • Easy to represent visually	• Cumbersome mathematics – generally requires automated cost estimating tool or macro to perform correctly • Prone to subjective judgment – many reasonable assumptions can be made about 'ramp up', 'steady state', and 'ramp down' periods

If the schedule specifies the rates and durations of these phases, then a Schedule-Based Method can be used; however, if the Trapezoid Method is used, there should be some historical, anecdotal, or quantitative basis for the rates and durations associated with each phase.

Method 4: Trapezoid Method Simulation Exercise

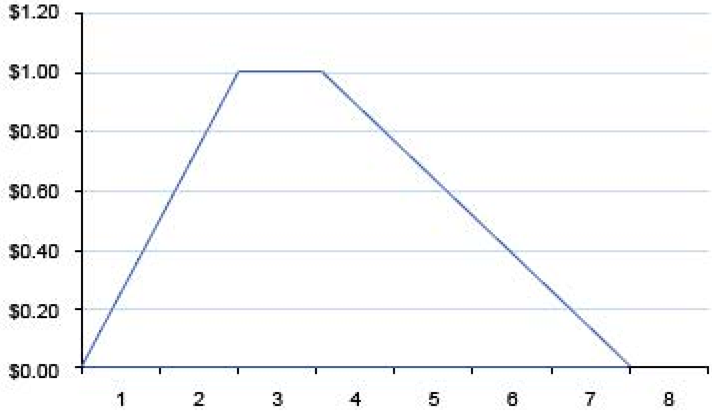
Variances in durations and rates can alter the results while using the Trapezoid Method. The combination of these factors, along with cost information, determine the shape of the trapezoid, and thus the profile of time phased cost.

A project office has plans that require 40 person-years with the following attributes – 2 years ramp up, with a one-year steady state and 4 year ramp down.

As the program gets under the way the program will be fully functional with a deployment date set - and as the program, winds down it will ramp down.

Total cost (FY 2010 $M):	$4.00
Cost per person (FY 2010$M):	$0.10
Implied number of person-years:	40
Years of ramp up:	2
Rate of ramp up (people/year):	5
Years of steady state:	1
Years of ramp down:	4

Year:	1	2	3	4	5	6	7	TOTAL
Total Cost (FY 2010$M):	$0.250	$0.750	$1.000	$0.875	$0.625	$0.375	$0.125	$4.000



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Long Description

Screen capture of the excel sheet that is used to calculate a trapezoid method. The user enters the following information -Total cost (FY 2010 $M), Cost per person FY 2010 $M, Rate of ramp up (people/year) and the excel sheet calculates the Total Cost (FY 2010 $M) over the specified number of FY and provides a total.

Knowledge Review

When the 'ramp up' phase and the 'ramp down' phase equals 0 years, to which method is the Trapezoid Method equivalent?

- [ ] Level-Loaded
- [ ] Base Year Throughput, with constant amounts in each year
- [ ] Schedule-Based, with constant quantities in each year
- [x] All of the above

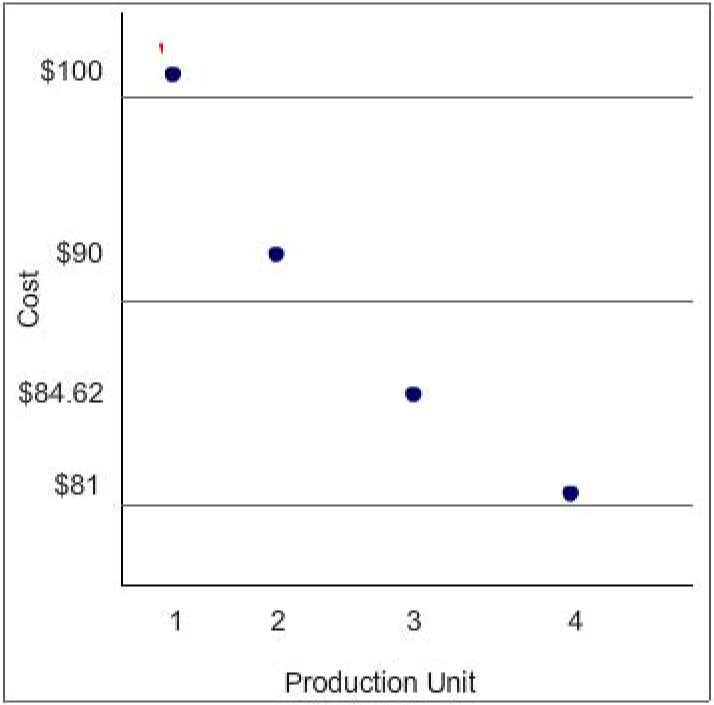
Check Answer

All of the above is correct. If there are neither 'ramp up' nor 'ramp down' phases, it is a 'steady state' phase which would give the trapezoid a rectangular shape. This is equivalent to the Level-Loaded Method, in which costs are constant between a given start year and end year. It is also equivalent to the Base Year Throughput, in the case that each base year amount is the same. Using a Schedule-Based Method, one could also replicate the same results by using constant (incremental) quantities in each year.

Method 5: Learning Curve Methods

The main idea behind learning curves is that cost declines at a constant rate each time production quantity doubles.

For example, consider a team of individuals responsible for building tracking units for an unmanned aerial system. As they continue to build the units, they establish efficiencies in production that help them complete the task quicker. As a result, more units can be produced with less effort and cost, as long as labor rates and material costs stay constant.



Cost, or effort, reduces at a constant rate each time production quantity doubles[1].

1. Adapted from Cost Estimating Body of Knowledge (CEBoK), Module 7, p. 6

D

Long Description

Animated graph/chart that shows increase in production and decrease in cost in direct relation to the example provided.

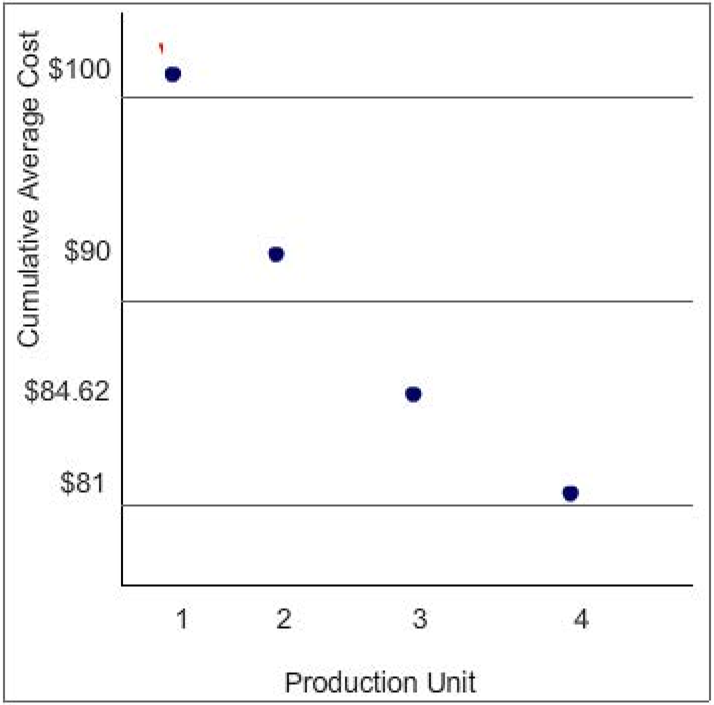
Wright Theory/Cumulative Average Unit Cost (CAUC) & Crawford Theory/Unit Theory (UT)

Wright Theory/Cumulative Average Unit Cost (CAUC)

In Wright, or CAUC theory, the cumulative average cost reduces at a constant rate each time production quantity doubles.

Crawford Theory/Unit Theory (UT)

In Crawford, or UT, the unit cost reduces at a constant rate each time production quantity doubles.



D

Long Description

Animated graph/chart that shows increase in production and decrease in cost in direct relation to the example provided.

CAUC Basic Equation

The CAUC basic equation for calculating learning curve is as follows:

$$y = ax^b$$

The components of the equation include the following:

Equation Element	Description
y	CAUC of first x units
a	Theoretical first unit cost (T1)
b	An indicator of learning curve slope (LCS); specifically, LCS = 2^b and b = ln(LCS)/ln(2)

The constant rate at which cost declines is (1-LCS).

CAUC LCS Example

Unit	FY	Est Cost (FY 2010 $K)
1	2010	300.00
2	2010	270.00
3	2011	253.86
4	2011	243.00
5	2011	234.90
6	2012	228.48
7	2012	223.18
8	2012	218.70
9	2012	214.82
10	-	-
11	-	-
12	-	-
13	-	-
14	-	-
15	-	-
Total		**2,186.94**

Popup Text

CAUC LCS Example

A 90% CAUC LCS implies that cumulative average unit cost is reduced by 10% each time production quantity doubles.

T1

Theoretical first unit cost. Typically used in the context of learning curve analysis.

Learning Curve Slope (LCS)

Learning Curve Slope (LCS) is defined as 100% minus the percentage by which cost is reduced each time production quantity doubles. For example, if cost is reduced by 10% each time production quantity doubles, this corresponds to an LCS of 90%.

UT Basic Equation

UT LCS Example

The UT theory also uses the same basic equation; however, the interpretation of x and y changes:

$$y = ax^b$$

The components of the equation include the following:

Equation Element	Description
y	Unit cost of x^{th} unit
a	Theoretical first unit cost (T1)
b	An indicator of LCS; specifically, LCS = 2^b and b = ln(LCS)/ln(2)

The constant rate at which cost declines is (1-LCS).

Unit	FY	Est Cost (FY 2010 $K)
1	2010	500.00
2	2010	450.00
3	2011	423.00
4	2011	405.00
5	2011	391.49
6	2012	380.79
7	2012	371.97
8	2012	364.50
9	2012	358.03
10	-	-
11	-	-
12	-	-
13	-	-
14	-	-
15	-	-
Total		**3,644.90**

Popup Text

UT LCS Example

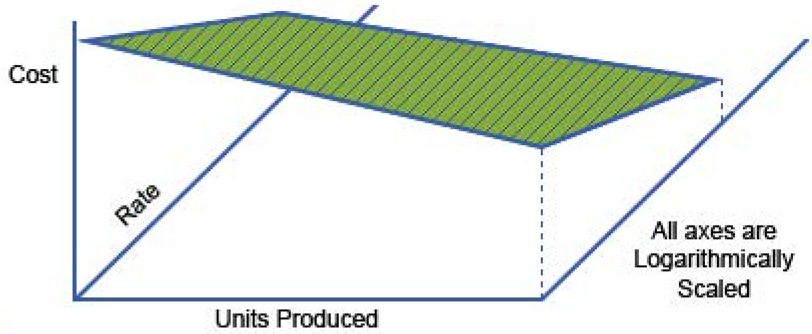
A 90% UT LCS implies that unit cost is reduced by 10% each time production quantity doubles.
In this case, LCS = 0.9. So b = ln(0.9)/ln(2) = -0.152.

Learning Curve Theory: UTRA

A popular variation of UT is Unit Theory with Rate Adjustment (UTRA).

This is equivalent to UT, except that there is (potentially) additional cost reduction due to rate variations. Specifically, cost also declines at a constant rate each time the production rate doubles.

A rate-adjusted method is potentially useful to model a situation in which economies of scale and benefits of automation are realized, such that higher rates of production imply lower unit costs.



Popup Text

Unit Theory with Rate Adjustment (UTRA)

Learning curve theory based on Unit Theory (UT), where the unit cost reduces at a constant rate each time production quantity doubles. However, an additional rate adjustment term is added, to reflect cost reduction associated with increased production rate (e.g. as in economies of scale).

UTRA Basic Equation

UTRA is equivalent to UT, except that additional terms indicate the cost reduction (if any) associated with rate adjustment, to reflect economies of scale:

$$y = ax^b r^{b2}$$

The components of the equation include the following:

Equation Element	Description
y	Unit cost of x^{th} unit
a	Theoretical first unit cost (T1)
b	An indicator of LCS. Specifically, LCS = 2^b and b = ln(LCS)/ln(2)
r	Rate of production
b2	Indicator of the rate slope (RS), similar to LCS, such that RS = 2^{b2} and b2 = ln (RS)/ln(2)

UTRA LCS Example

Popup Text

UTRA LCS Example

If LCS = 95% and RS = 90%, then cost is reduced by 5% when production quantity doubles, and is reduced by 10% when production rate doubles.

Application of Learning Curve Concepts to Time Phasing

The Learning Curve Theory is relevant to time phasing based on the following:

- When learning is thought to be involved, the Cost Estimating Relationship (CER) output is typically T1, or the cost of some other theoretical unit.
- Learning Curve Theory may be used to estimate the cost of the remaining units.
- Because multiple units might be procured in lots, with different lots being procured in different years, the learning curve assumptions dictate the time phased profile of cost.

In the cases of UT and UTRA, it may also be necessary to calculate or assume a lot midpoint (LMP).

Popup Text

Cost Estimating Relationship (CER)

Cost Estimating Relationship (CER) is a mathematical relationship that defines cost as a function of one or more parameters such as performance, operating characteristics, physical characteristics, etc.

Lot Midpoint (LMP)

- The LMP is the "representative unit" from each lot: the unit at which the average cost of the lot occurs[1]
- The true LMP depends on b, which may not be known when the time phasing algorithm is set up
- The "best LMP" heuristic[2], also known as the Algebraic Lot Midpoint, is the average of the arithmetic and geometric means of the index numbers of the first unit (FU) and last unit (LU):

$$LMP = \frac{FU + LU + 2\sqrt{FU \cdot LU}}{4}$$

In Excel: LMP = AVERAGE(AVERAGE(FU,LU),GEOMEAN(FU,LU))

- If b is known, then the exact value of LMP is $(\Sigma i^b/N)^{1/b}$, where i runs from FU to LU, and N is the number of units in the lot[3]

References:

1. Cost Estimating Body of Knowledge (CEBoK), Module 7, p. 39
2. Nussbaum, Dan. Evaluation of Alternative Estimators of Learning Curve Lot Midpoints. Society of Cost Estimating and Analysis (SCEA) Journal of Cost Analysis (Spring, 1994)
3. Cost Estimating Body of Knowledge (CEBoK), Module 7, p. 39

Learning Curve Methods Simulation Exercise

D

Long Description

A movie with a three by seventeen table which depicts unit number, fiscal year and estimated cost in thousands of dollars.

Suppose that the first unit cost (T1) of the receiver/exciter (REX) portion of a radar is thought to be $100K. Given a production schedule of 2 units during the first year, 3 in the following year, and 4 in the final year, what are the time phased costs?

Step 1 adds the instruction: "Specify the learning curve assumptions to be used." It also adds the caption: "In this example, assume a 90% UT curve with no rate adjustments." while highlighting the first 9 rows of the table.

Step 2 adds the instruction: "Determine the year of each unit." It also adds the caption: "Fiscal Years are assigned to units based on the specified production schedule." while each fiscal year appears.

Step 3 adds the instruction: "Estimate the cost of each unit." It adds the caption: "The cost of each unit is estimated using the equation y = ax b , where a is T1 ($100K), y is the cost of the xth unit, and b = ln(.9)/ln(2)." while highlighting the Cost column in rows 1 through 9. It also adds the caption: "The total cost of 9 units is $728.98K, which is significantly lower than the implied total cost of $100K/unit * 9 units = $900K that would have been estimated in the absence of learning." while highlighting the Total row.

Step 4 adds the instruction: "Calculate time phased costs." It adds the caption: "The final step is to simply aggregate the costs by year, in order to create a time phased profile of costs. Note that each year's costs are determined by two factors: the quantity of units in that year, and how far 'down the curve' the year occurs. In this example, 2012 has twice as many units as 2010, but does not nearly have twice the cost, due to being farther 'down the learning curve.'" while showing the calculated yearly totals.

Learning Curve Methods Simulation Exercise, Cont.

Use the Learning Curve Calculator to the right, which allows you to change key learning curve and other assumptions, such as quantities by year, LCS, and T1, in order to view the impact of these changes on time phased costs.

Download the Learning Curve Spreadsheet.

Learning Curve Calculator

REX T1 Cost (FY 2010) ($K):	$ 100
2010 Quantity	2
2011 Quantity	3
2012 Quantity	4
UT LCS:	90 %
	Update

Unit	FY	Est. Cost (FY 2010) ($K)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
Total		

Learning Curve Strengths and Limitations

Strengths (+)	Limitations (-)
A number of advantages and strengths are apparent when using Learning Curve Methods: • Learning Curve Theory accounts for cost reduction due to learning, economies of scale, and/or dispersion of fixed costs over a greater number of units explicitly in the time phasing method. • It is a well-documented, quantifiably justified basis for hypothesis of learning (first asserted by T.P. Wright, 1936; it is from this proposal that CAUC curves are sometimes known as "Wright" curves). • Finally, time phased costs are not directly *assumed*, but are driven by other data or *assumptions*. This may make the analysis more credible, as the analyst does not assume the very thing that he/she needs to calculate (time phased costs). For example, when "a 90% UT learning curve" is assumed, the relative portion of cost in each year is not explicitly stated.	The main drawback of Learning Curve Methods is that they introduce an additional degree of uncertainty into the estimate. Even if the underlying CER perfectly estimates T1 (or the cost of some other unit), the analyst must also specify the appropriate learning theory and slope in order for the results to be credible. This is why the collection of actual LCS data from analogous programs is important when using these types of methods.

Knowledge Review

Assuming that costs cannot be negative, what are the theoretical lower limits on LCS for CAUC and UT?

- [ ] 0% for CAUC, 50% for UT
- [x] 50% for CAUC, 0% for UT
- [ ] 0% for both
- [ ] 50% for both

Check Answer

The correct answer is **50% for CAUC, 0% for UT**. If costs cannot be negative, then the minimum possible unit cost is \$0. If T1 is \$0, then no learning can take place, so it must be assumed that T1 > \$0. If T1 > \$0, but T2 = \$0, then under CAUC, the cumulative average cost of the first two units is (T1+0)/2 = T1/2, which is 50% of the "average" through one unit (T1), so a 50% LCS is implied. For UT, if T2 = \$0, then this represents a 100% reduction relative to T1, which corresponds to an LCS of 0%.

Knowledge Review

Which of these learning curves assumes more cost reduction, in percentage terms?

- [x] 90% CAUC curve
- [ ] 90% UT curve
- [ ] Equal amounts of cost reduction
- [ ] Depends on the T1 assumed in each case

Check Answer

A **90% CAUC curve** implies that T2 = 0.8*T1, because the average cost of the first two units, (T1+T2)/2, need to equal 90% of T1. In this instance, (T1+T2)/2 = (T1 + 0.8*T1)/2 = 1.8*T1/2 = 0.9*T1, as required. On the other hand, a 90% UT curve implies that T2 = 0.9*T1, by definition. Thus, cost is reduced by 20% with the CAUC curve and by 10% with the UT curve having the same LCS. The CAUC curve implies relatively more cost reduction.

Knowledge Review

For what value of b are the Learning Curve Methods equivalent to the Schedule-Based Method?

- [x] 0
- [ ] 1
- [ ] 2
- [ ] 3

Check Answer

If **b = 0**, then LCS = 2^b = 2^0 = 1. The amount of cost reduction is (1-LCS), so an LCS of 1 implies no cost reduction, which would be equivalent to using schedule inputs (with constant unit cost) as the basis for time phasing.

Method 6: Probability Distribution-Based Methods: Overview

The final group of time phasing methods we will discuss is called the Probability Distribution-Based group of methods. In theory, it is possible to fit any historical time phased cost profile to a Probability Distribution, and use those same distributional parameters to predict future time phased costs method. There are two main types of distributions:

- Beta and PERT Beta: These use variants of the Beta Distribution to approximate time phased cost
- Rayleigh and Weibull: These use variants of the Weibull Distribution to approximate time phased costs

Both types of distribution allow for "front loading" of effort, where cost starts low, then peaks, then levels off.

These methods are typically used to time phase the costs associated with development efforts, particularly when the underlying technology is immature.

Beta and PERT Beta

The first group of Probability Distribution-Based methods on which we will focus are related to the Beta function. The Beta Distribution is a well-defined Continuous Distribution, based on the Beta function, whose properties can be used to time phase a previously estimated cost over several years. As stated before, the primary utility of this type of method lies in time phasing costs associated with development efforts.

The PERT Beta Distribution is either an approximation or special case of the Beta Distribution, depending on how one interprets its parameters. PERT Beta has the advantage that its parameters have a "real world" interpretation: minimum, most likely, and maximum values. For this reason, it is used extensively in project management to estimate task durations. While schedule estimating and the related tasks of estimating schedule durations are related to time phasing, they are not *methods* of time phasing in the context of cost analysis, and therefore are beyond the scope of this CLM.

When PERT Beta is used as an approximation of the Beta Distribution, it is not always a great approximation. In fact, the relative discrepancies in the mean and variance between the PERT Beta and Beta Distributions having identical low, most likely, and high values can be rather large.

Properties of the Beta Distribution

Let us distinguish further between the Beta and PERT Beta Distributions. The Beta Distribution is a very flexible one. The Beta Distribution also has finite lower and upper bounds. For purposes of this discussion, those bounds are assumed to be 0 and 1, because percentage of time phased cost is always between 0% and 100%. It can take on virtually any shape that a cost analyst can imagine.

The Beta Distribution is characterized by two shape parameters, alpha and beta (α and β). Its mean and variance are defined in terms of those parameters. It is relatively straightforward to "calibrate" alpha and beta based on historical data, so that is where the Beta Distribution has its greatest utility. However, it has limited utility in the context of using expert opinion, as alpha and beta have no readily explainable "real world" interpretation.

A Beta Distribution has the greatest utility when analogous historical data are available, from which best-fitting alpha and beta can be derived.

Popup Text

Any Shape

A Beta Distribution is flexible and takes on any shape. These include:

- Linear (including horizontal)
- Narrow and wide, regular and inverted U-shapes
- Left or right skewed
- Right skewed when $\alpha < \beta$; left skewed when $\alpha > \beta$
- Can resemble a Triangular Distribution when $\alpha = 1$ and $\beta = 2$ (or vice versa)
- Can resemble a Normal Distribution when $\alpha = \beta = 4$

Mean and Variance

Mean (μ) = $\alpha/(\alpha+\beta)$

Variance ($\sigma2$) = $\alpha\beta/[(\alpha+\beta)2\ (\alpha+\beta+1)]$

Properties of the PERT Beta Distribution

The PERT Beta Distribution can be viewed as either a special case or approximation of the Beta Distribution. It is used extensively in critical path analysis by project managers, wherein experts estimate low, most likely, and high durations for each task or activity. However, in the context of cost estimating, these three parameters are interpreted as start year (a), end year (b), and year of peak, or "most," expenditure (m).

The PERT Beta Distribution also has a mean and variance defined in terms of its parameters. The mean is simply a weighted average of the three parameters, with m getting 4 times the weight of the other two parameters. The standard deviation is simply the difference of the high and low parameters, divided by 6; the variance is the square of that number.

$$\text{Mean} = (a+4m+b)/6 \qquad \text{Variance} = (b-a)^2/36$$

Each PERT Beta Distribution implies a unique Beta Distribution, but many Beta Distributions can have identical PERT properties (i.e. have identical start year, peak year, and end years). The distributions are equivalent in only two cases, as documented in the cited article.[1]

The PERT Beta Distribution is most useful when there is a paucity of reliable historical or analogous data, because subject matter experts can be asked for parameters with easy-to-explain "real world" interpretations, such as start year, end year, and year of peak expenditure.

Has greatest utility when little or no analogous historical data are available, because experts can be asked for low, high, and most likely values.

Reference: 1. Keefer, Donald L. and Bodily, Samuel E. (1983). Three-point Approximations for Continuous Random variables. Management Science 29(5), pp. 595-609.

Application of Beta Distribution to Time Phasing

Now that we have seen the fundamental differences between the Beta and PERT Beta Distributions, let us look at examples of each of them applied to time phasing, starting with the Beta Distribution. For this example, suppose that our analogous program took 7 years, and cost $719.5K. The costs were incurred as shown in this table.

We will use that data to determine the "best fitting" Beta Distribution parameters, then use those parameters to estimate the time phased costs associated with a "new" program.

YEAR	COST	CUM COST
1	$105,000	$105,000
2	$215,000	$320,000
3	$185,000	$505,000
4	$135,000	$640,000
5	$55,000	$695,000
6	$23,000	$718,000
7	$1,500	$719,500

Beta Distribution Example

Here, the fitted values of the Beta distribution are approximately 2 for alpha and 4 for beta. Whenever alpha is less than beta, as is the case here, the distribution will be right skewed. This behavior is characteristic in development efforts, which tend to be "front-loaded." In fact, note that both the actual and fitted cost data imply that more than 70% of the cost is incurred after less than half the time has elapsed.

YEAR	COST	CUM COST	% OF COST	PRED % OF CUM COST	PRED % OF COST	ERROR (PCT PTS)		alpha	2.0141
1	$105,000	$105,000	14.6%	15.1%	15.1%	-0.5%		beta	4.0284
2	$215,000	$320,000	29.9%	44.2%	29.1%	0.8%			
3	$185,000	$505,000	25.7%	71.1%	26.9%	-1.2%		SSE:	0.00045
4	$135,000	$640,000	18.8%	89.0%	17.9%	0.9%			
5	$55,000	$695,000	7.6%	97.5%	8.5%	-0.8%			
6	$23,000	$718,000	3.2%	99.8%	2.4%	0.8%			
7	$1,500	$719,500	0.2%	100.0%	0.2%	0.0%			

Other Ways to Fit a Distribution

There are many ways to fit a distribution to a set of data, and there is a good deal of software available to aid in this process. In this simple example, the analyst generates Beta distribution-based predictions of cumulative cost, and uses those to generate predictions of time phased cost. Those predictions are compared to actual time phased cost, and the values of alpha and beta are adjusted via an optimization routine (in this case, Excel Solver) so that some objective function (in this case, sum of squared error) is minimized.

Knowledge Review

Which of the following is **NOT** a main advantage of using the Beta Distribution, as opposed to the PERT Beta Distribution?

- [ ] Flexible
- [ ] Easy to specify, using historical data
- [x] Easy to specify, using expert opinion
- [ ] None of these

Check Answer

Because alpha and beta have no real world interpretation, the Beta Distribution is **not easy to use when the inputs come from expert opinion**. In these cases, the PERT Beta is more suitable.

Knowledge Review

Whenever alpha is ______ than beta, the distribution will be _____.

- [ ] When alpha is greater than beta, the distribution will be right skewed.
- [ ] When alpha is less than beta, the distribution will be left skewed.
- [x] When alpha is less than beta, the distribution will be right skewed.
- [ ] When alpha is less than beta, the distribution will be equal on the left and right sides.

Check Answer

When alpha is less than beta, the distribution will be right skewed.

Rayleigh/Weibull

The next family of distributions we will discuss is the Weibull family. We will focus on the generalized Weibull Distribution, and also the Rayleigh (sometimes called Norden-Rayleigh) distribution, which is a special case of the Weibull.

The Weibull Distribution is a continuous probability distribution with positive parameters k (*shape*) and λ (*scale*).

1. Equivalent to an Exponential Distribution when k=1
2. Equivalent to a Rayleigh Distribution when k=2

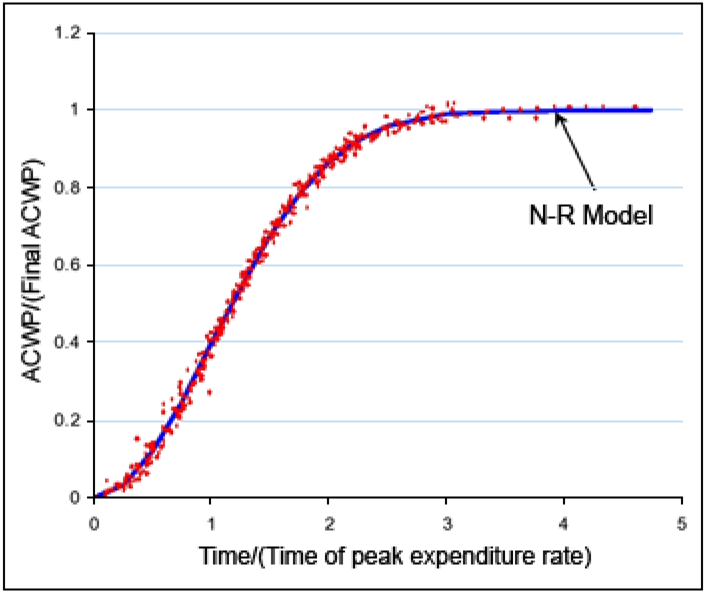
As we will see, both distributions have been shown to be effective in estimating time phased development costs. Rayleigh curves are extensively used time phasing estimates of software development costs.[1,2] But since Rayleigh curves are also Weibull curves, the same comment applies to Weibull curves as well. In addition, generalized Weibull Distributions have been shown to be superior to both Rayleigh curves and traditional EVM formulas in time phasing of ship construction costs.[3]

References:

1. Coleman, Richard et. al. Rayleigh Curves: A Tutorial.
2. Lee, Dr. David et. al. The Rayleigh Analyzer; Norden-Rayleigh Analysis.
3. Rudolph, Shawn. Ship Construction Estimates at Completion: A New Technique Using the Weibull Function.

Why Norden-Rayleigh Distributions?

Let's now address the Rayleigh Distribution specifically. It is sometimes called the Norden-Rayleigh Distribution because Peter Norden is credited with first proposing the use of a Rayleigh Distribution to model software development costs. As the graphic shows, the Norden-Rayleigh, or N-R model, is a very good predictor of final cost, based on a set of 385 actual cost of work performed (ACWP) reports from 36 programs in a certain database.



As can be seen from the graph, the Rayleigh Distribution has (or can have) the characteristic form of "front loading," where expenditures are low in the beginning (for example, during the design, planning, and requirements specification phase) then peak (during code and unit test), then decline during the latter phases of the project (say, during perfective maintenance). Note that the graphic shows cumulative, not incremental cost, so the horizontal portion of the curve on the right side corresponds to near zero expenditure.

Resource: Lee, Dr. David. Norden-Rayleigh Analysis: A Useful Tool for EVM in Development Projects. Presented at the College of Performance Management Conference: Washington, DC (2002).

D

Long Description

A line graph with the x axis labeled Time/(Time of peak expenditure rate) with the values of 0 to 5 in increments of 1 and a y axis of ACWP/(Final ACWP) with the values of 0 to 1.2 in increments of .2.

Why Weibull Distributions?

Rudolph (2010) shows that generalized Weibull Distributions tend to be better predictors of time phased cost than specific Rayleigh Distributions in the case of ship construction. In particular, Weibull Distributions are more **flexible**:

- Scale and shape parameters can vary
  - Rayleigh: scale parameter only; shape parameter = 2 in all cases
- Can have inflection point prior to mode
  - Rayleigh: no inflection point prior to mode
- The area underneath the curve to the left of the mode (i.e. % of development costs incurred prior to the peak) can vary
  - Rayleigh: this value is always 0.393

We will see examples of both Rayleigh and Weibull curves on future pages.

Knowledge Review

Which distribution(s) have historically been associated with which types of development efforts? (Select all that apply)

- [x] A. Rayleigh for software; Weibull for ship construction
- [ ] B. Weibull for software; Rayleigh for ship construction
- [x] C. Both for software; Weibull for ship construction
- [ ] D. Beta for software; Rayleigh for ship construction

Check Answer

The Rayleigh Distribution has been proposed for software development, while the Weibull Distribution has been proposed for ship construction, which is answer "**A**." However, because Weibull Distributions are also Rayleigh Distributions, answer "**C**" is also acceptable.

Knowledge Review

Suppose that a particular set of historical time phased costs exactly matched a Rayleigh Distribution. What would be the best fitting distribution for that data?

- [ ] Beta
- [ ] Rayleigh
- [ ] Weibull
- [x] Rayleigh and Weibull would fit equally well

Check Answer

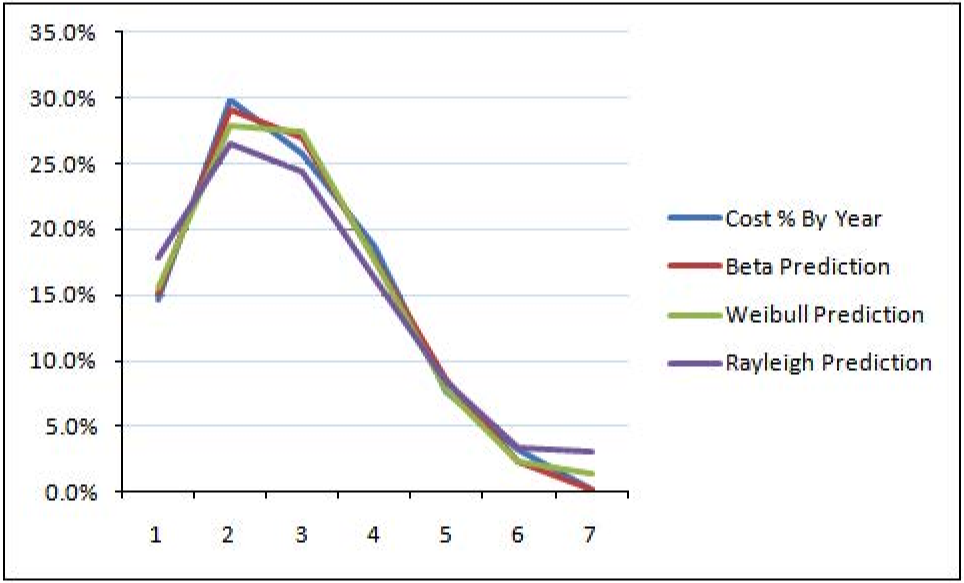
Because a Weibull Distribution is also a Rayleigh Distribution, **Rayleigh and Weibull would fit equally well.**

Comparison of Relative Fits

The Beta, Weibull, and Rayleigh Distributions were calibrated based on historical data and all fit that data reasonably well.

They all have the general shape we would expect in software development or ship construction efforts.

All three replicate the time profile of that data with reasonable fidelity.



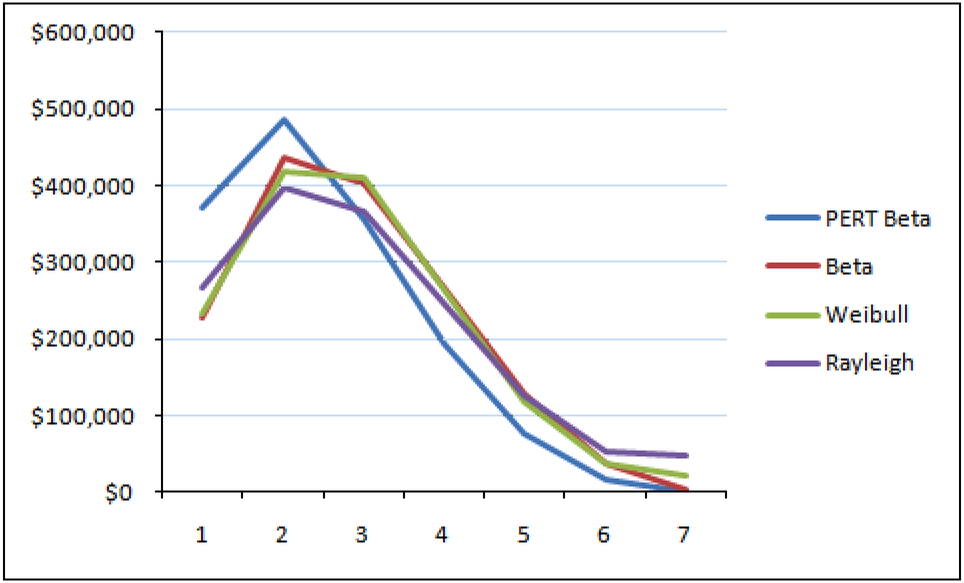
D

Long Description

A line graph with the x axis of years with the values of 0 to 7 in increments of 1 and a y axis of percentages with the values of 0 to 35% in increments of 5%. The following distributions are plotted on the line graph. Cost % by year, Beta Prediction, Weibull Prediction and Rayleigh Prediction.

Comparison of Future Predictions

We also used a fourth distribution, PERT Beta, to predict the time phased costs of a hypothetical future program.



The main difference is that the PERT Beta's parameters were informed by subject matter expert opinion, rather than by historical data.

In any case, all four distributions give projected cost profiles of the same general shape. Note that total cost is constant across distributions.

So if a distribution is relatively lower than the others in a certain area, it is higher in other areas to "compensate" such that the areas underneath all four curves are equivalent.

In summary, we fit three different distributions (Beta, Weibull, and Rayleigh) to historical data. All three of them fit reasonably well. In this example, the Beta Distribution provides the best fit, but this result does not necessarily hold in general. The analyst should keep an open mind with respect to alternative distribution forms. In fact, the hypothesized distributions need not be limited to these three.

D

Long Description

A line graph with the x axis of years with the values of 0 to 7 in increments of 1 and a y axis of dollars with the values of 0 to $600,000 in increments of $100,000. The following distributions are plotted on the line graph Pert Beta, Beta, Weibull, and Rayleigh.

Summary

This concludes the lesson on Time Phasing Methods. Here's a recap of the key topics covered in this lesson:

1. Time phasing a cost estimate is just that—time phasing an existing cost estimate. It is not the same as cost estimating. The practice of time phasing assumes, or presupposes, the existence of a credible cost estimate that must be spread over a number of years. After it is spread, one can apply the appropriate escalation factors, create schedule estimates, formulate budgets, etc. but none of these are actually part of the time phasing process itself.
2. There are many methods by which a number can be spread across multiple years. Hence, time phasing methods can range from very simple to very complex. Examples of simple methods include Level-Loaded (where base year costs are constant in each year) and Throughput (in which the analyst already has costs by year, obviating the need for a distinct time phasing method). Examples of complex methods include Probability Distribution-Based Methods, where the stochastic properties of various distributions determine the relative spreads of cost.
3. The appropriate type of method to use depends upon the nature of the problem that the cost analyst is trying to solve. There is no "one size fits all rule," but certain types of cost elements generally lend themselves to certain types of time phasing approaches.

Lesson Completion

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